Introduction to Structural Equations

Statistics for Psychosocial Research II

Structural Models

October 30, 2006

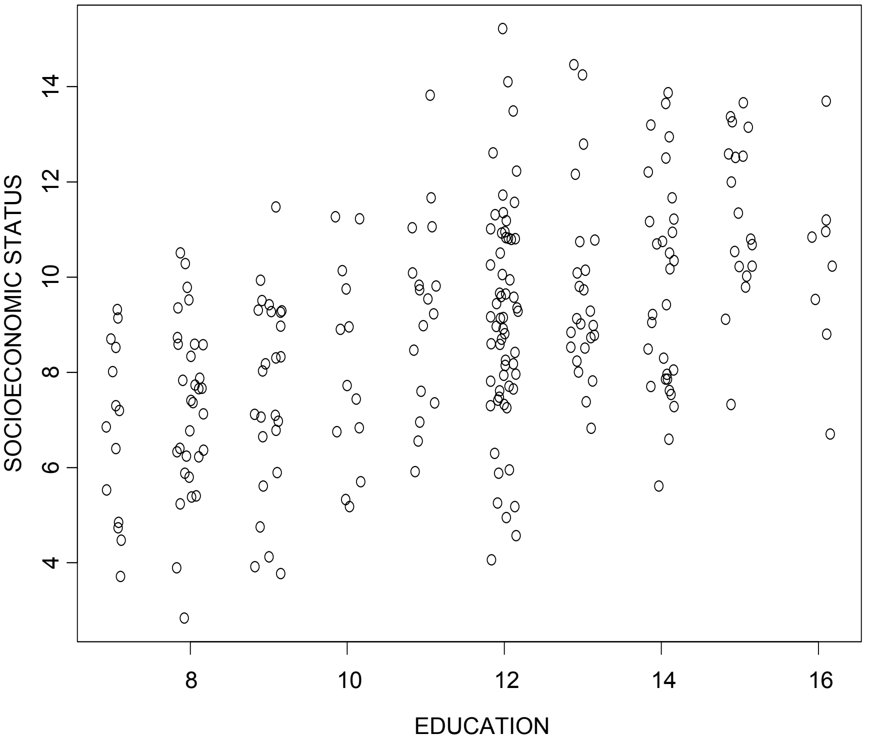
Course Overview

(1) Structural Regression/Path Analysis
  (a) “effect mediation” versus “controlling for”
  (b) causality

(2) Regression plus measurement structures from last term
  (a) if we ignore measurement, “item regression”
  (b) factor analysis: structural equations with latent variables
  (c) latent class analysis: latent class regression

General Idea

- How does outcome vary with predictors?
- Make inference on hypothesis about how predictors affect outcome
- Predict individual outcomes



Challenge

- How do we measure latent outcomes (and predictors)?
- There are multiple responses
- Approach 1:
  - $Y_1,\ldots,Y_n$ measure the same thing. Treat individually or summarize Y's.
- Approach 2:
  - Call ideal outcome η
  - If we <u>knew</u> η, then $\eta_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \ldots$
  - But we don't know it:
    - infer η from factor analysis or latent class analysis
    - regress η on X's

Three approaches to assessing association between covariates and multiple responses

(1) Summarize then analyze (STA)

(2) Analyze then summarize (ATS)

(3) Summarize AND analyze: (SAA)

- Structural Equations
- 2 parts
  - measurement component
  - structural/regression component

Example: Depression Study
Summarize then Analyze (STA)

- Clinical trial of two anti-depressants
- Which anti-depressant is more effective for treating depression?
- Depression symptoms were based on the Hamilton Depression Rating Scale (HAM-D).

17 Symptoms

- Depressed mood
- Guilt feelings
- suicide
- Insomnia (x3)
- Work and activities
- Psychomotor retardation
- agitation
- anxiety
- Somatic symptoms
- …..

For each item, write the correct number on the line next to the item. (Only one response per item)

_____1. DEPRESSED MOOD (Sadness, hopeless, helpless, worthless)
0= Absent
1= These feeling states indicated only on questioning
2= These feeling states spontaneously reported
3= Communicates feeling states non-verbally—i.e., through facial expression, posture, voice, and tendency to weep
4= Patient reports VIRTUALLY ONLY these feeling states in his spontaneous verbal and non-verbal communication
_____2. FEELINGS OF GUILT
0= Absent
1= Self reproach, feels he has let people down
2= Ideas of guilt or rumination over past errors or sinful deeds
3= Present illness is a punishment. Delusions of guilt
4= Hears accusatory or denunciatory voices and/or experiences threatening visual hallucinations
_____3. SUICIDE
0= Absent
1= Feels life is not worth living
2= Wishes he were dead or any thoughts of possible death to self
3= Suicidal ideas or gesture
4= Attempts at suicide (any serious attempt rates 4)
_____4. INSOMNIA EARLY
0= No difficulty falling asleep
1= Complains of occasional difficulty falling asleep—i.e., more than ½ hour
2= Complains of nightly difficulty falling asleep
_____5. INSOMNIA MIDDLE
0= No difficulty
1= patient complains of being restless and disturbed during the night
2= Waking during the night—any getting out of bed rates 2 (except for purposes of voiding)
_____6. INSOMNIA LATE
0= No difficulty
1= Waking in early hours of the morning but goes back to sleep
2= Unable to fall asleep again if he gets out of bed

Example: Summarize then Analyze (STA)

- <u>Summarize</u>:
  - Add up the number of symptoms, or “score” the HAM-D.
  - Treat the score as “fixed” or “observed” outcome.
  - But, we know better! It is not measured perfectly.
  - What is the reliability of the HAM-D???
- <u>Analyze</u>: See how the outcome relates to predictor (i.e., treatment)

Summarize Then Analyze

1. Sum up HAM-D score pre and post and take difference:

   Pre-treatment score: $Y_{i1} = Y_{i1,1} + Y_{i1,2} + \cdots + Y_{i1,21}$

   Post-treatment score: $Y_{i2} = Y_{i2,1} + Y_{i2,2} + \cdots + Y_{i2,21}$

   Difference: $D_i = Y_{i2} - Y_{i1}$

2. Evaluate association with Y_i and treatment

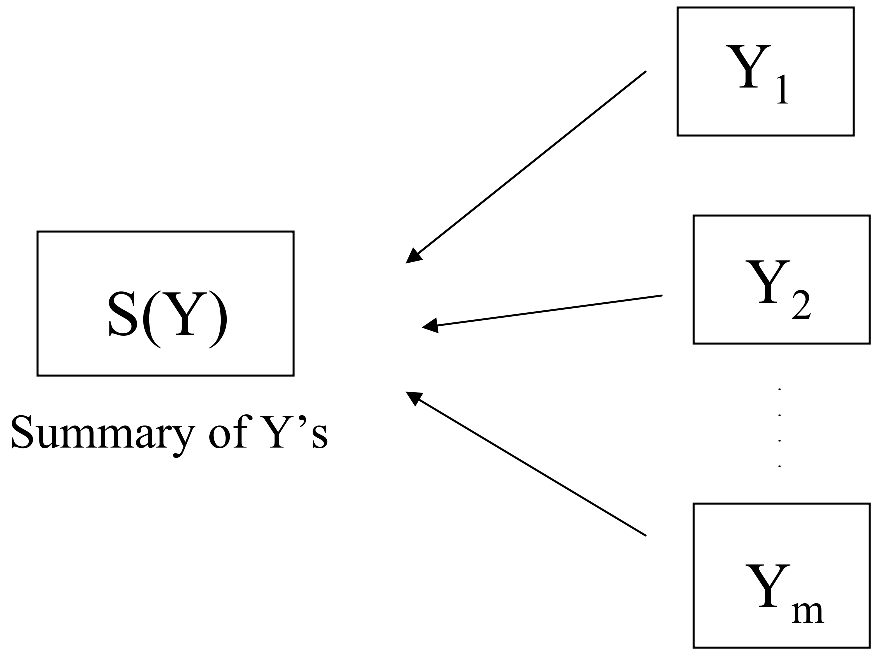
$$D_i = \beta_0 + \beta_1 trt_i$$

   where trt_i = 1 of treatment A, and 0 if treatment B

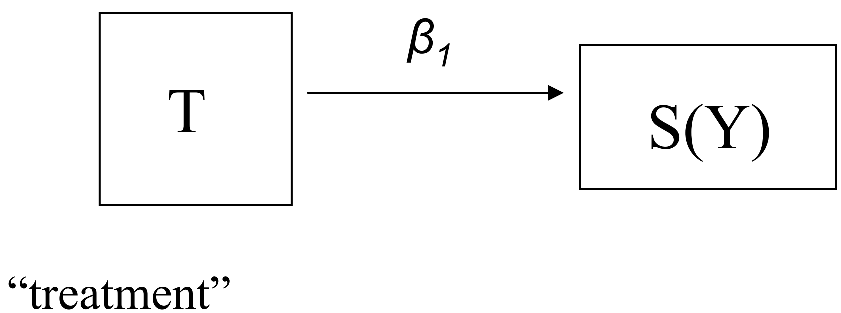
3. Make inference about treatment effect based on β_1

STA: Two models estimated separately

Model 1:



Model 2:



STA: so what is the problem???

- We are ignoring that S(Y) is measured with error.
- Note that that S(Y) has reliability less than 1.
- In our example: S(Y) represents an “imperfect measure” of depression with reliability of about 0.88 (depending on population).
- Aren’t we then overestimating the variation in our outcome by using S(Y)?
- Recall: $Var(T_x) < Var(X)$
- What effect might that have on the standard error of β_1?

Another Approach: Analyze Then Summarize (ATS)

1. Analyze: for each of the 21 items in the HAM-D, see if treatment is associated with improvement.

1. Define outcome per item:

$$D_{i,1} = Y_{i2,1} - Y_{i1,1}$$

$$\vdots$$

$$D_{i,21} = Y_{i2,21} - Y_{i1,21}$$

2. Estimate association per item with treatment:

$$D_{i,1} = \beta_{0,1} + \beta_{1,1} trt_i$$

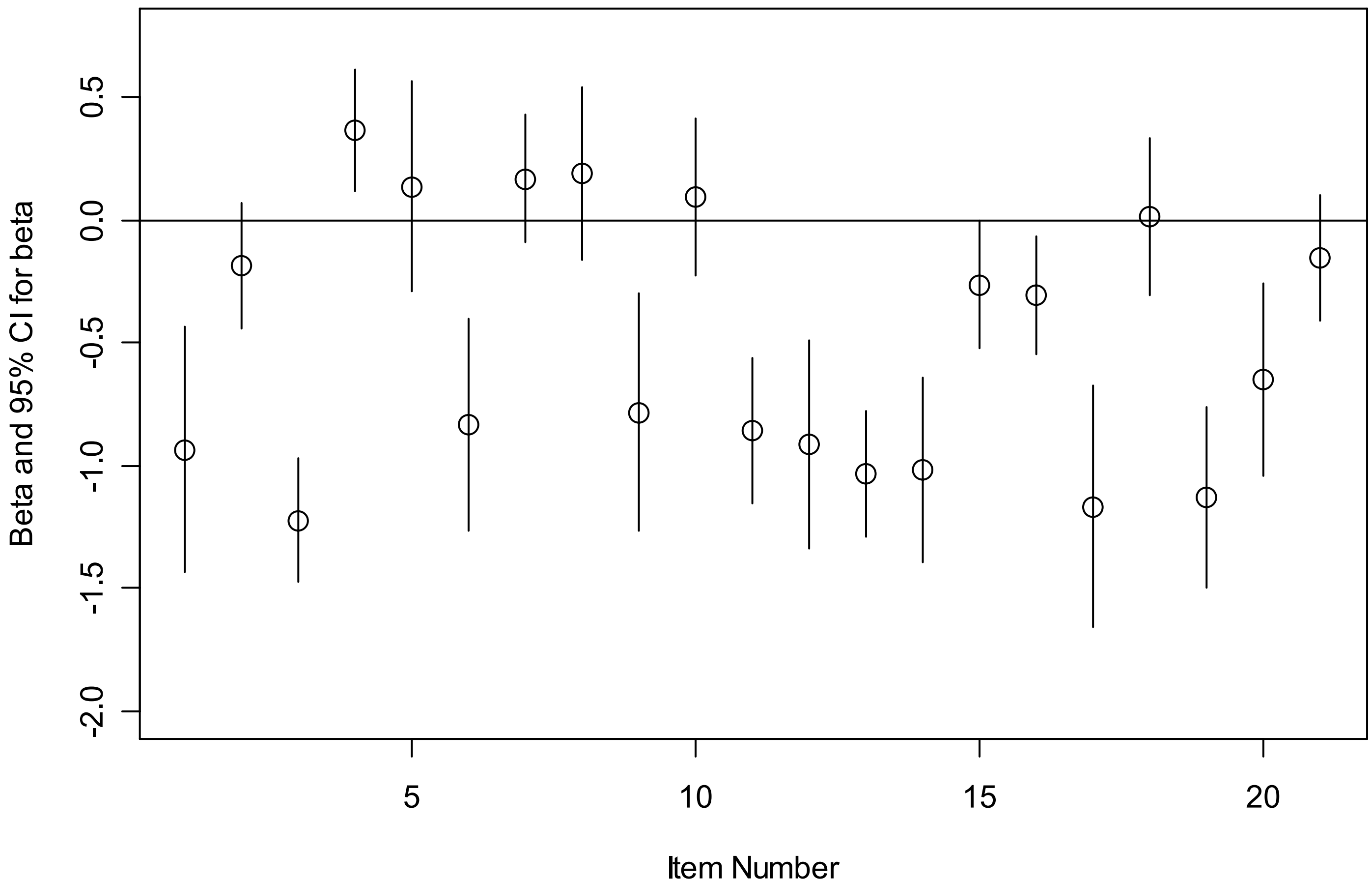
$$D_{i,2} = \beta_{0,2} + \beta_{1,2} trt_i$$

$$\vdots$$

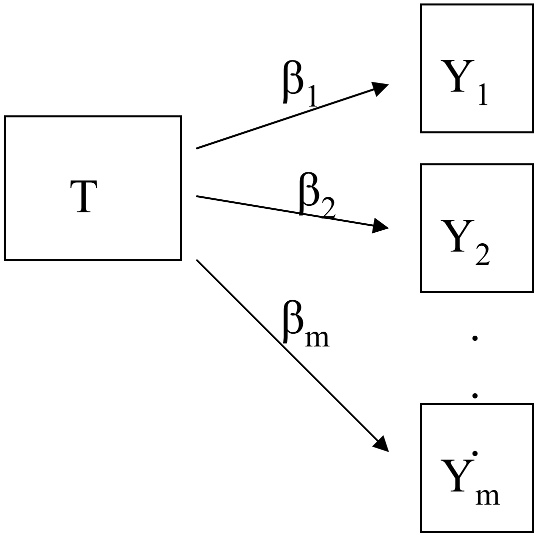
$$D_{i,21} = \beta_{0,21} + \beta_{1,21} trt_i$$

Another Approach: Analyze Then Summarize (ATS)

2. Summarize: Qualitatively or quantitatively evaluate the associations



Analyze then Summarize



Fit *m* regressions to individually describe association between T and each Y.

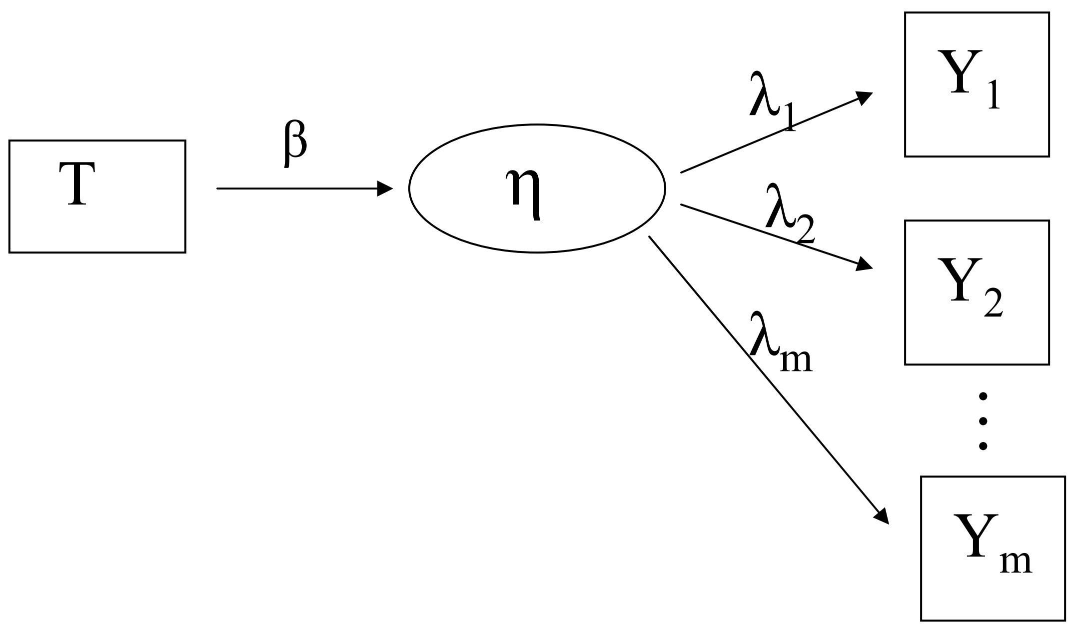
Then summarize associations.

So what is wrong with ATS?

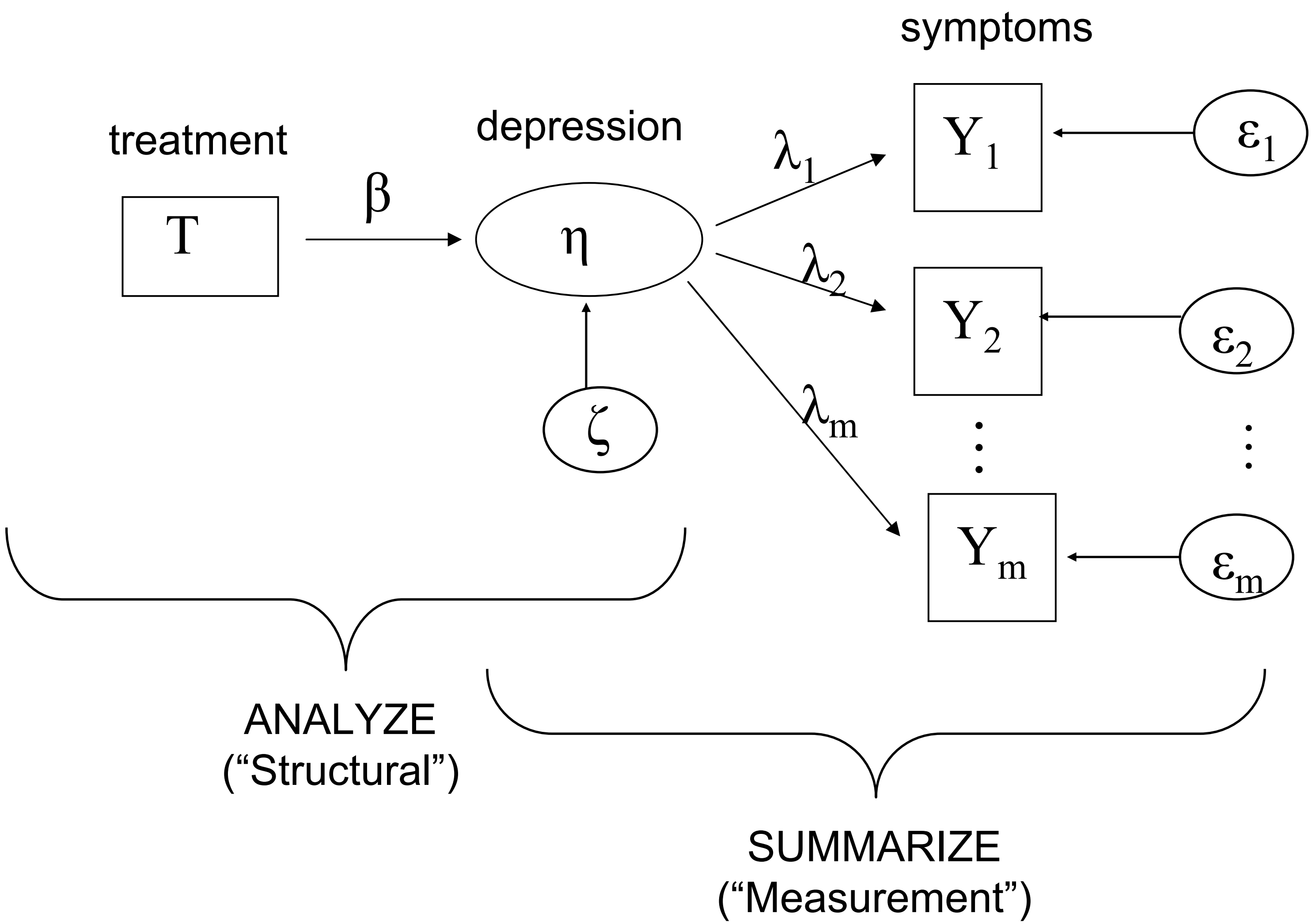
- How do we answer the question: “Which treatment works better?”
- We get individual answers.
- Often hard to summarize after the analysis has been done.
- (More about this in ‘Item Regression lecture’)

Summarize and Analyze Simultaneously (SAA)

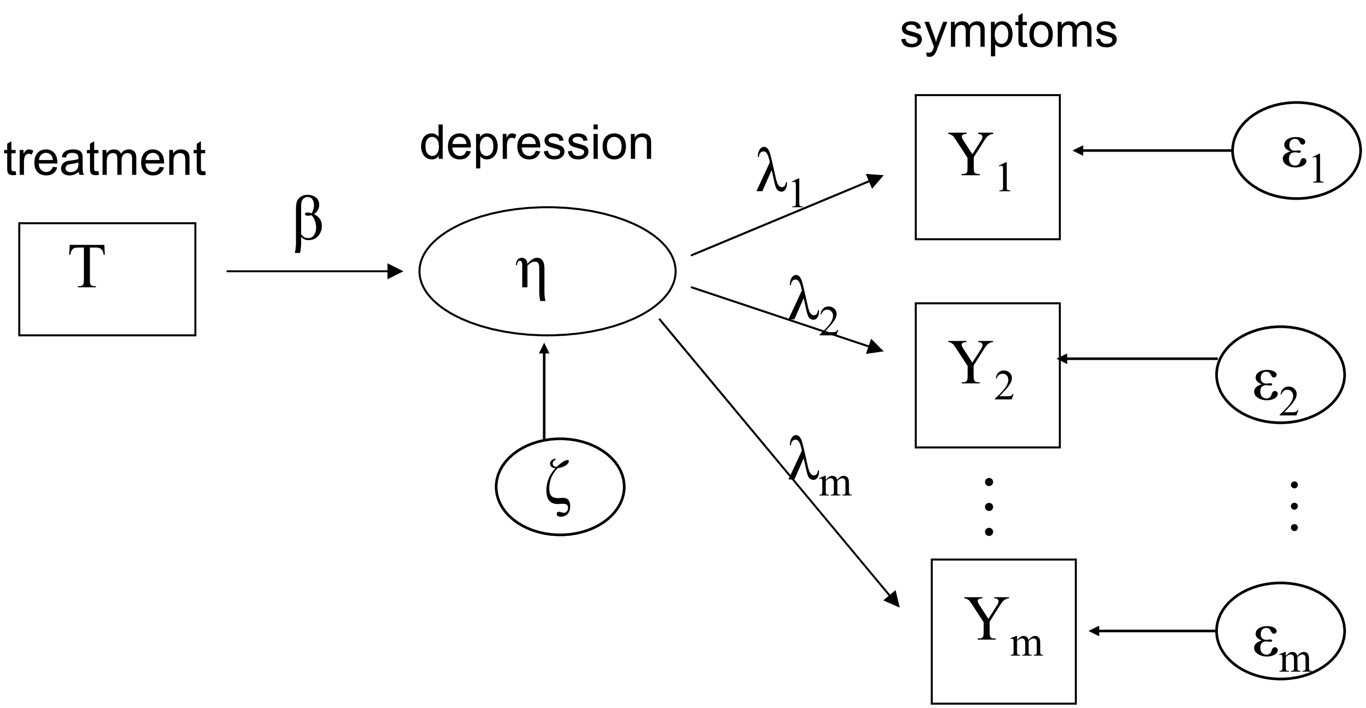
- Fit ‘summarize’ and ‘analyze’ components at the same time.
- One big model
- Accounts for measurement error of latent variable



Summarize and Analyze Simultaneously



Summarize and Analyze Simultaneously



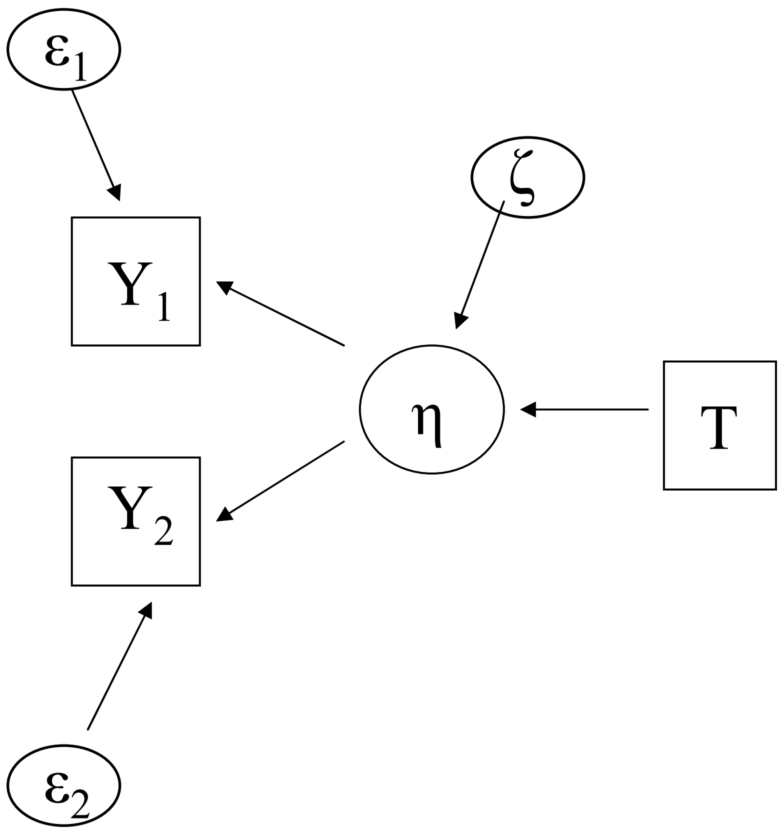
Example: $\eta = \beta X + \zeta$

$$Y_1 = \lambda_1 \eta + \varepsilon_1$$
$$Y_2 = \lambda_2 \eta + \varepsilon_2$$
$$\vdots$$
$$Y_m = \lambda_m \eta + \varepsilon_m$$

Caveat

- When is analyze then summarize better?
- What if some treatment affects some of the symptoms but not all of them?
- What does that imply about the measurement?

Path Notation

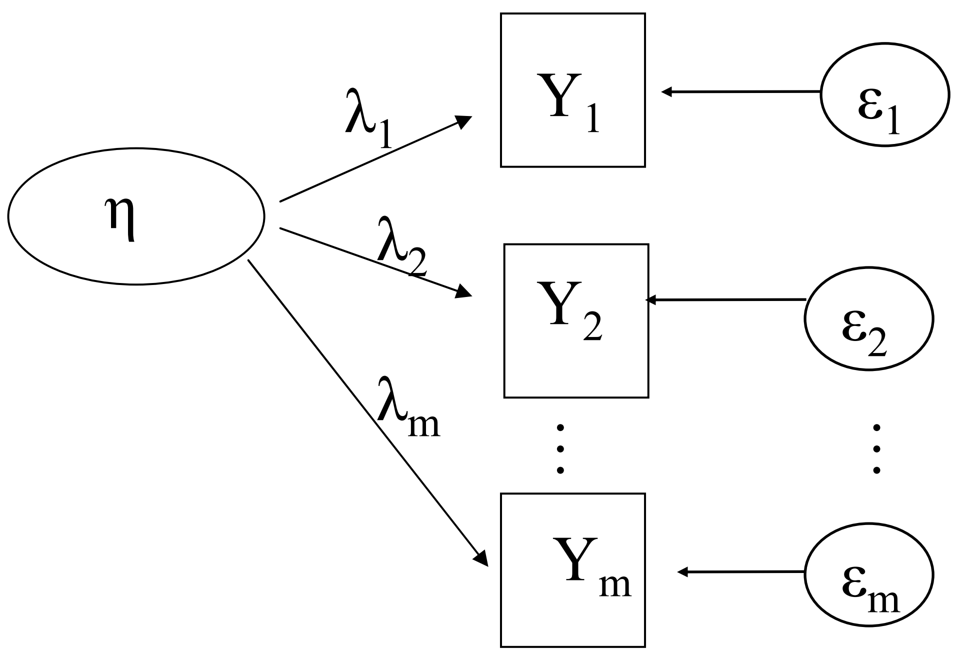


- Relationships
  - straight arrow (causal)
  - curved arrow (unspecified)
- Variables
  - circles vs. squares
  - exogenous (independent)
  - endogenous (dependent)
- Errors
  - one for every endogenous variable
  - unexplained component of predicted variables

Components of Structural Equation Model

(A) Measurement Piece

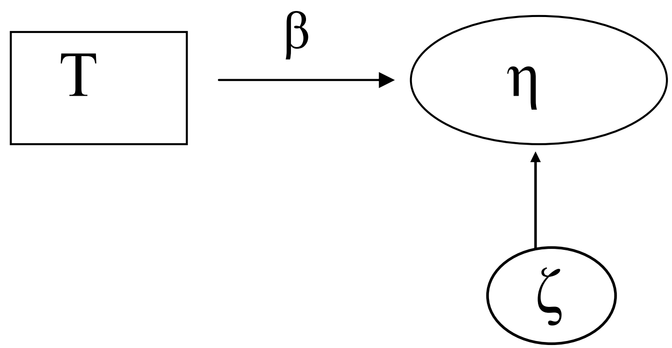
- how latent variable related to “surrogates”
- comprised of η’s and Y’s



Components of Structural Equation Model

(B) Structural Piece

- how latent variable is related to its predictors
- regression
- comprised of η's and T

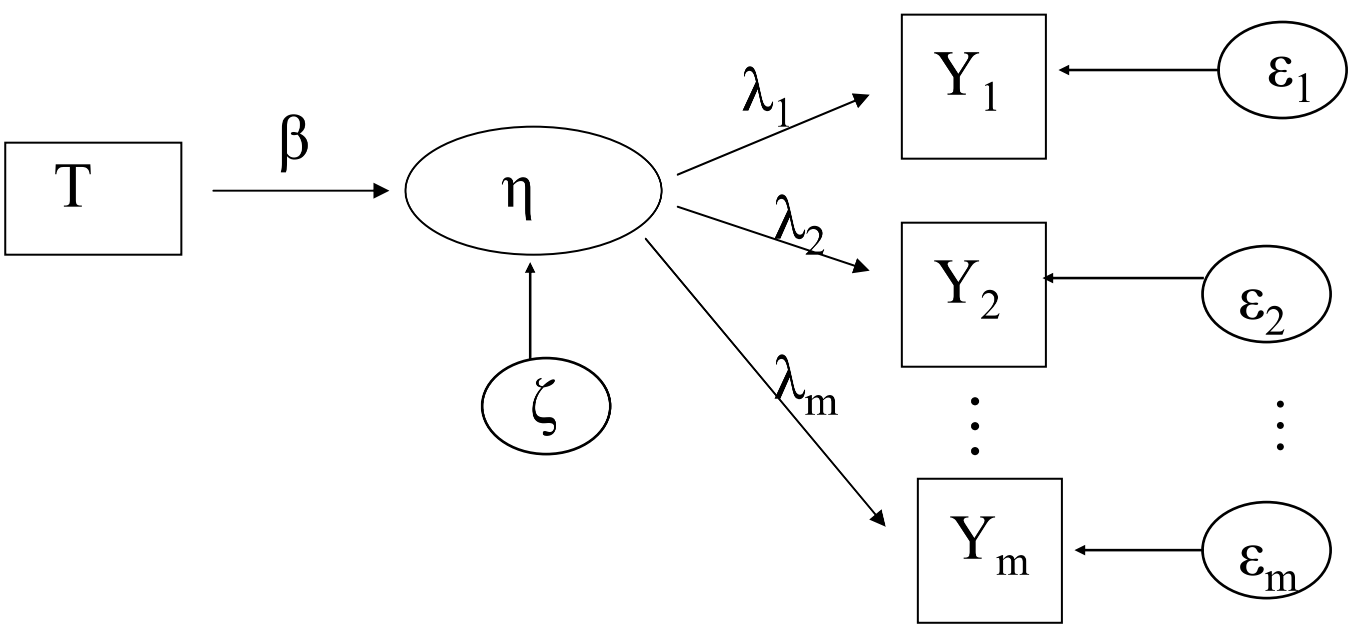


Components of Structural Equation Model

(C) Both components are fit in ONE step

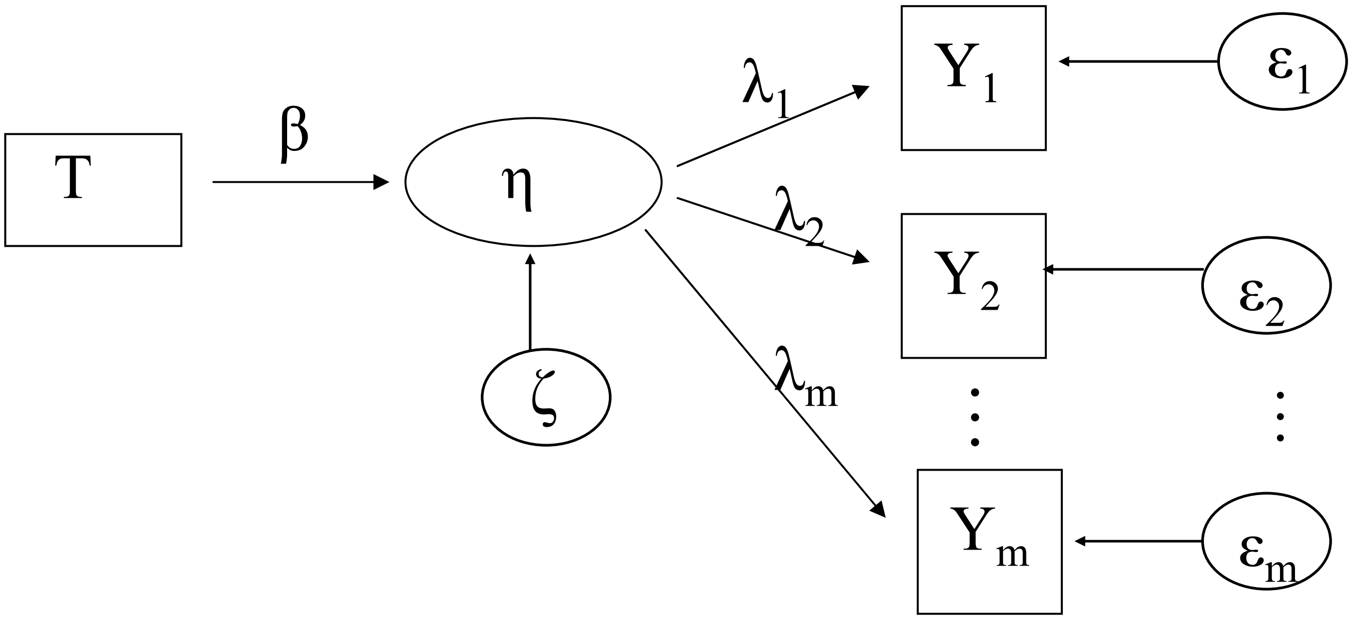
Why better? Does not assume η (i.e., "summary" of Y's) known, which acknowledges measurement error.

Why bad? If model is misspecified, then inference is misleading.



Statistical way of considering relationship between T and Y

$$P(Y = y \mid T) = \sum_{r=1}^{R} P(Y = y, \eta = r \mid T)$$
$$= \sum_{r=1}^{R} P(Y = y \mid \eta = r, T) P(\eta = r)$$



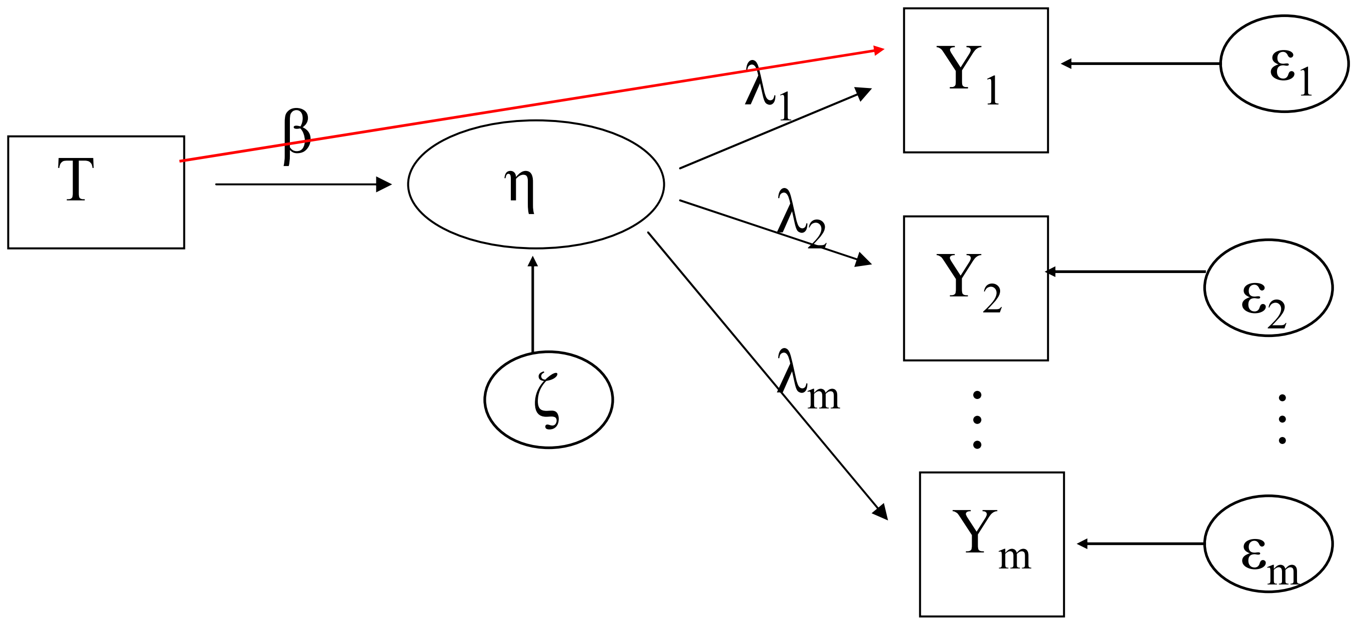
Assumption 1: Non-Differential Measurement

Equivalent interpretations:

- covariates do not predict observed responses after controlling for latent status
- no arrows between T and Y's
- Y and T independent given η

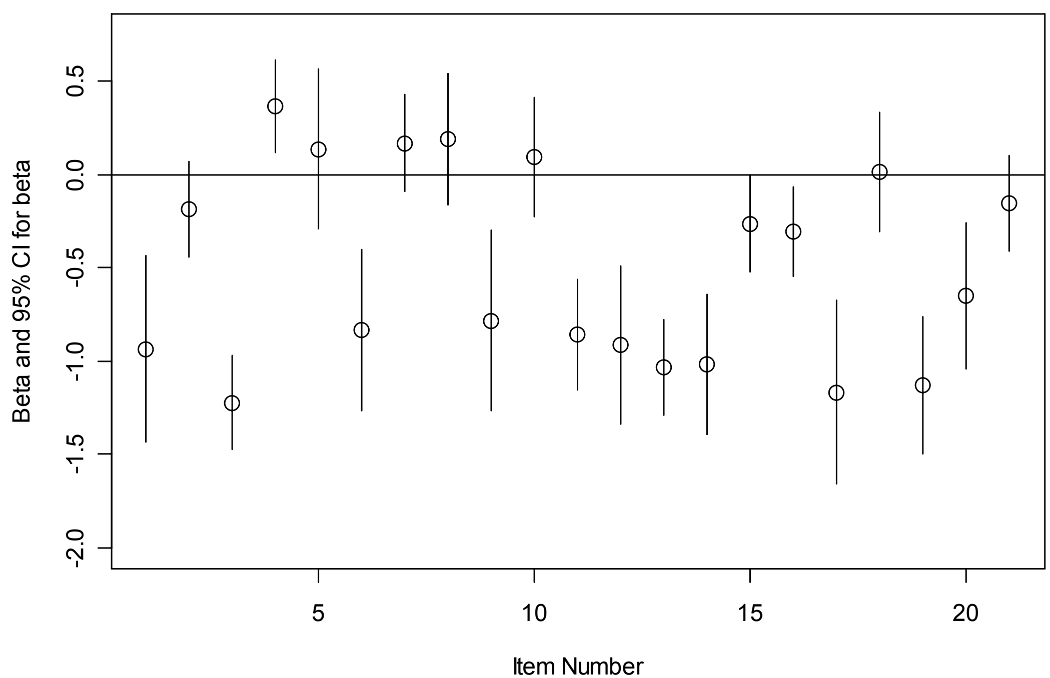
$$P(Y = y \mid \eta, T) = P(Y = y \mid \eta)$$

NOT OK UNDER NON-DIFFERENTIAL MEASUREMENT:



HAM-D Depression Example

- Does treatment affect the “depression” or symptoms?
- Implications for “differential measurement”?



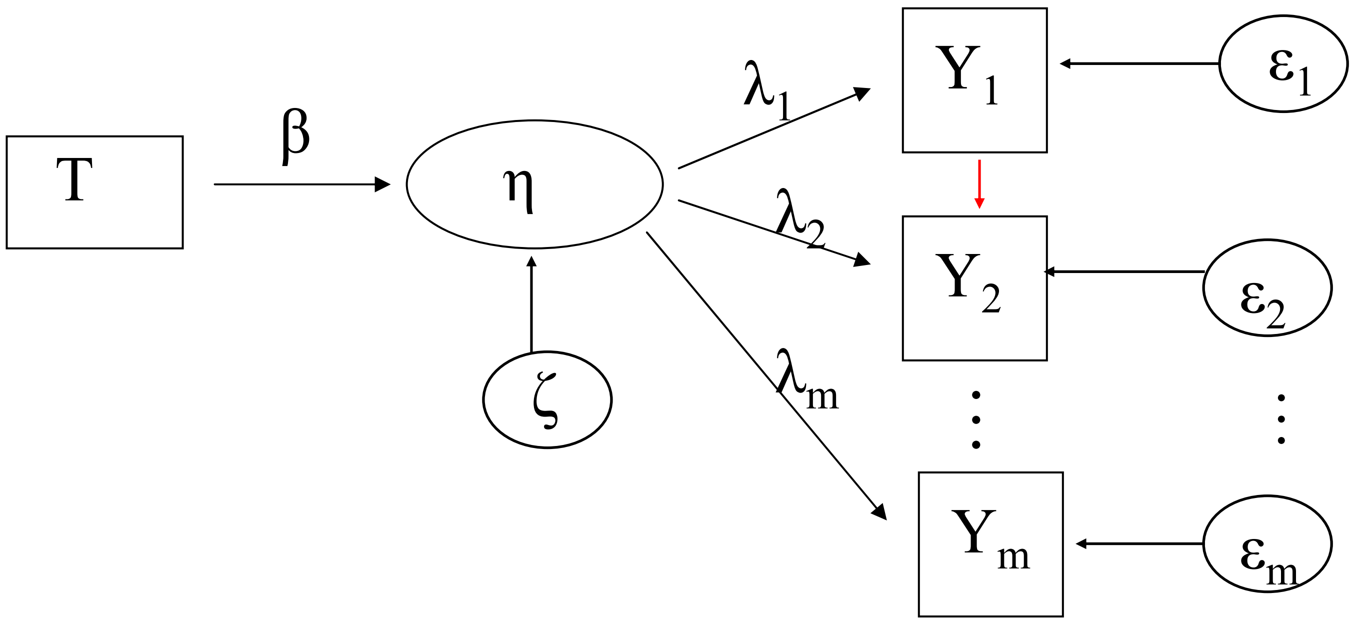
Assumption 2: Local/Conditional Independence

Equivalent Interpretations

- latent variable explains all association between observed variables
- no arrows among measurement errors
- observed variables are independent given η

$$P(Y_1 = y_1, Y_2 = y_2 \mid \eta) = P(Y_1 = y_1 \mid \eta) P(Y_2 = y_2 \mid \eta)$$

NOT OK UNDER CONDITONAL INDEPENDENCE:



NOT OK UNDER CONDITIONAL INDEPENDENCE: