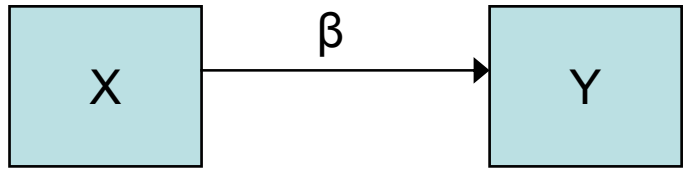
Final Thoughts on SEMs

Statistics for Psychosocial Research II:
Structural Models
November 27, 2006

Outline for today

- Brief discussion of Standardized Coefficients
- Latent Variables in SEM
  - Adding regressors
  - Relating LVs
    - Indexes
    - Factors
  - Stratification by binary variable
- Some estimation issues to consider
- If we have time: a couple of examples.

Standardized Coefficients



- Make “relative” direct influences clearer
- Standardized coefficient of β:

$$\beta^* = \beta \frac{\sigma_x}{\sigma_y}$$

- The standardized effect is the mean change in standard deviation units of Y for a one standard deviation change in X.
- “If X increases by one standard deviation, then we expect that Y will increase by β^* standard deviations.”
- Standardized can be easier for making inferences from SEMs
- Standard errors on the standardized are generally not correct (estimated using the correlation matrix)

Quick Little Derivation

Model: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + e$

Standardizing Variables

$$x_1^* = x_1 / \sigma_{x_1}; \quad x_2^* = x_2 / \sigma_{x_2}; \quad y^* = y / \sigma_y$$

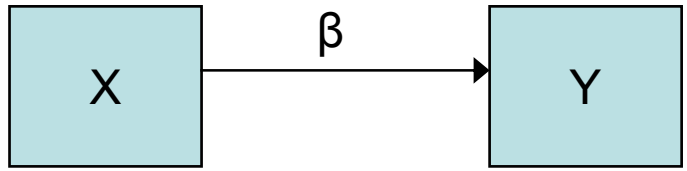
$$\sigma_{x_1} x_1^* = x_1; \quad \sigma_{x_2} x_2^* = x_2; \quad \sigma_y y^* = y$$

Substitute standardized variables into model:

$$\sigma_y y^* = \beta_0 + \beta_1 \sigma_{x_1} x_1^* + \beta_2 \sigma_{x_2} x_2^* + e$$

$$y^* = \frac{\beta_0}{\sigma_y} + \beta_1 \frac{\sigma_{x_1}}{\sigma_y} x_1^* + \beta_2 \frac{\sigma_{x_2}}{\sigma_y} x_2^* + e$$

Standardized Coefficients



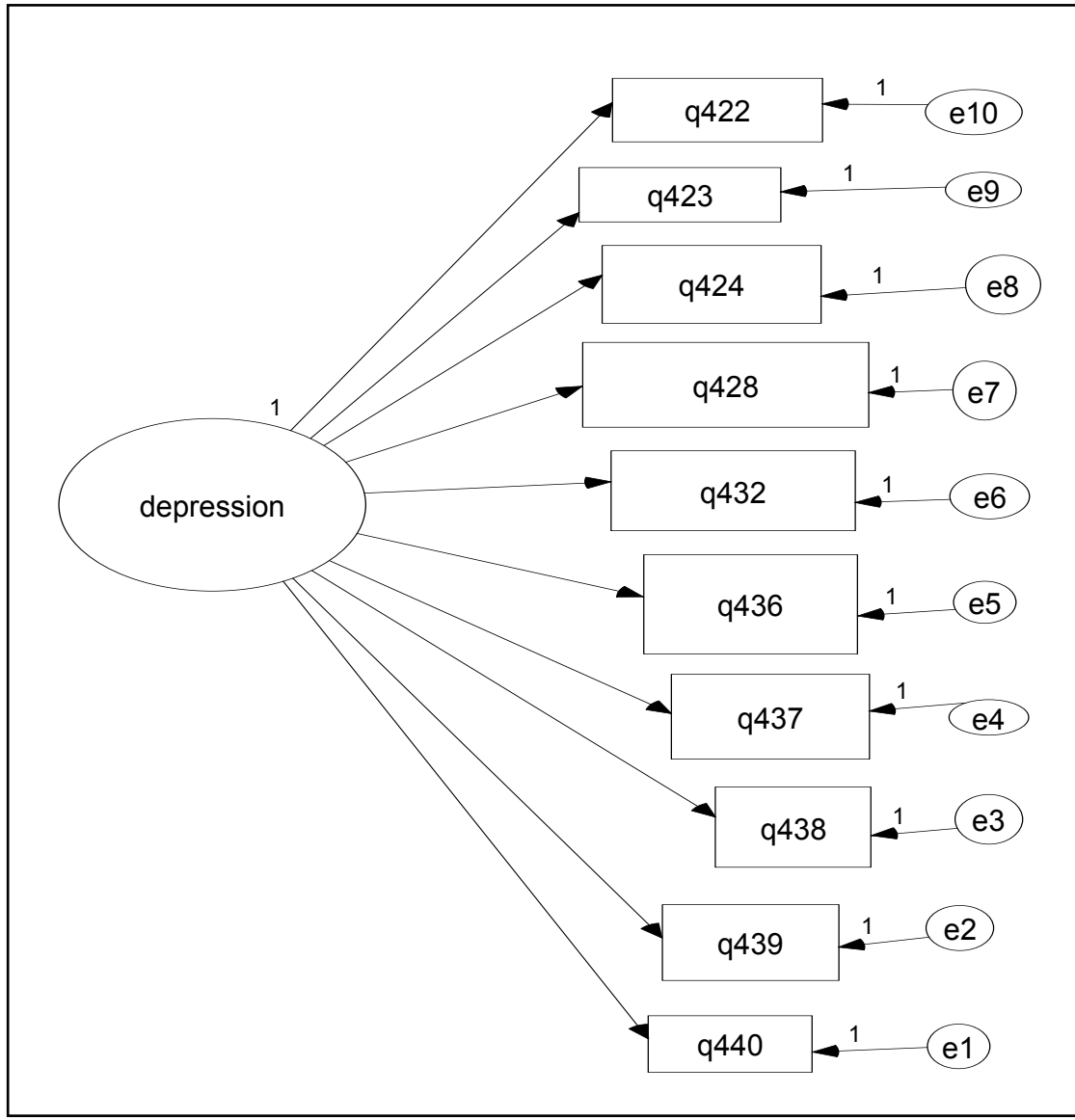
- Caveats:
  - Testing statistical significance of standardized effects is messy.
    - Standard deviations take different values in different datasets
    - The distribution of the standardized β depends on the standard deviation so that standard errors depend on them.
  - Beware of comparing standardized coefficients for the same variable across groups!
    - Standardized effects might be different due to different standard deviations
    - Look at unstandardized coefficients when comparing across groups.
  - Programs (e.g. AMOS vs. EQS) are inconsistent in how they define “standardized

Latent Variables in SEM

- Much like path analysis with observed variables
- Some additional considerations

Example: Depression

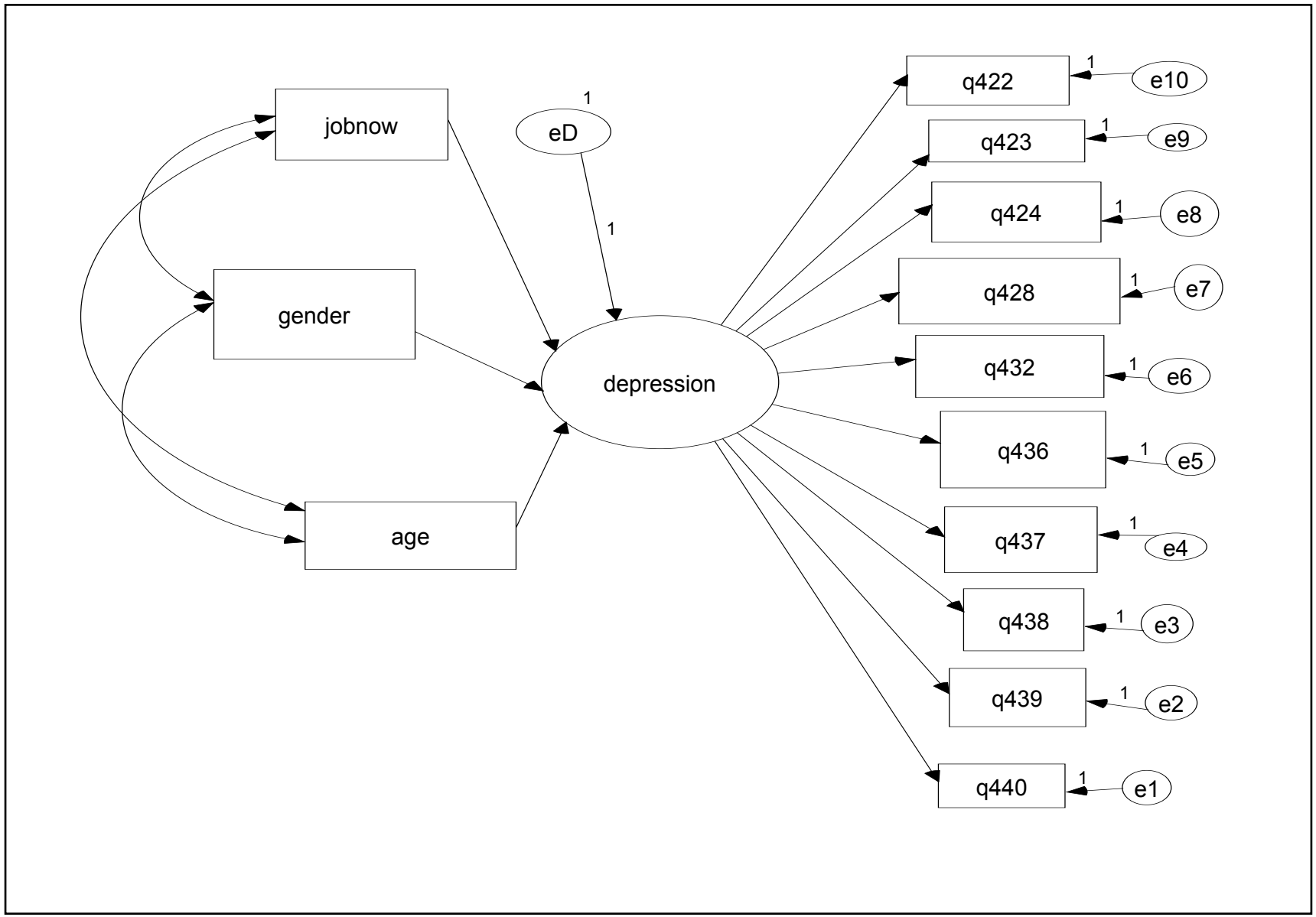
- concentrate
- happy
- energetic
- useful
- enjoy activities
- depressed
- confidence
- worthless
- hopeless
- worried



Constraining Latent Variables

- CFA model: 2 options
  - set variance of (exogenous) LV equal to 1 (my preference)
  - set path coefficient to one of its indicators equal to 1. This scales the LV in the same units as the indicator.
- More complicated models:
  - endogenous LV
    - set variance of its error term to 1
    - set path coefficient to one of its indicators to 1.
- MUST do one of these or model is not identifiable!

Adding Regressors



Structural Equation

measurement model piece:

depression and its symptoms

$$y_k = \lambda_k \eta + \varepsilon_k, \quad k = 1,\ldots,10$$

regression piece:

age, gender, job predict depression

$$\eta = \gamma_1 age + \gamma_2 gender + \gamma_3 job + \zeta$$

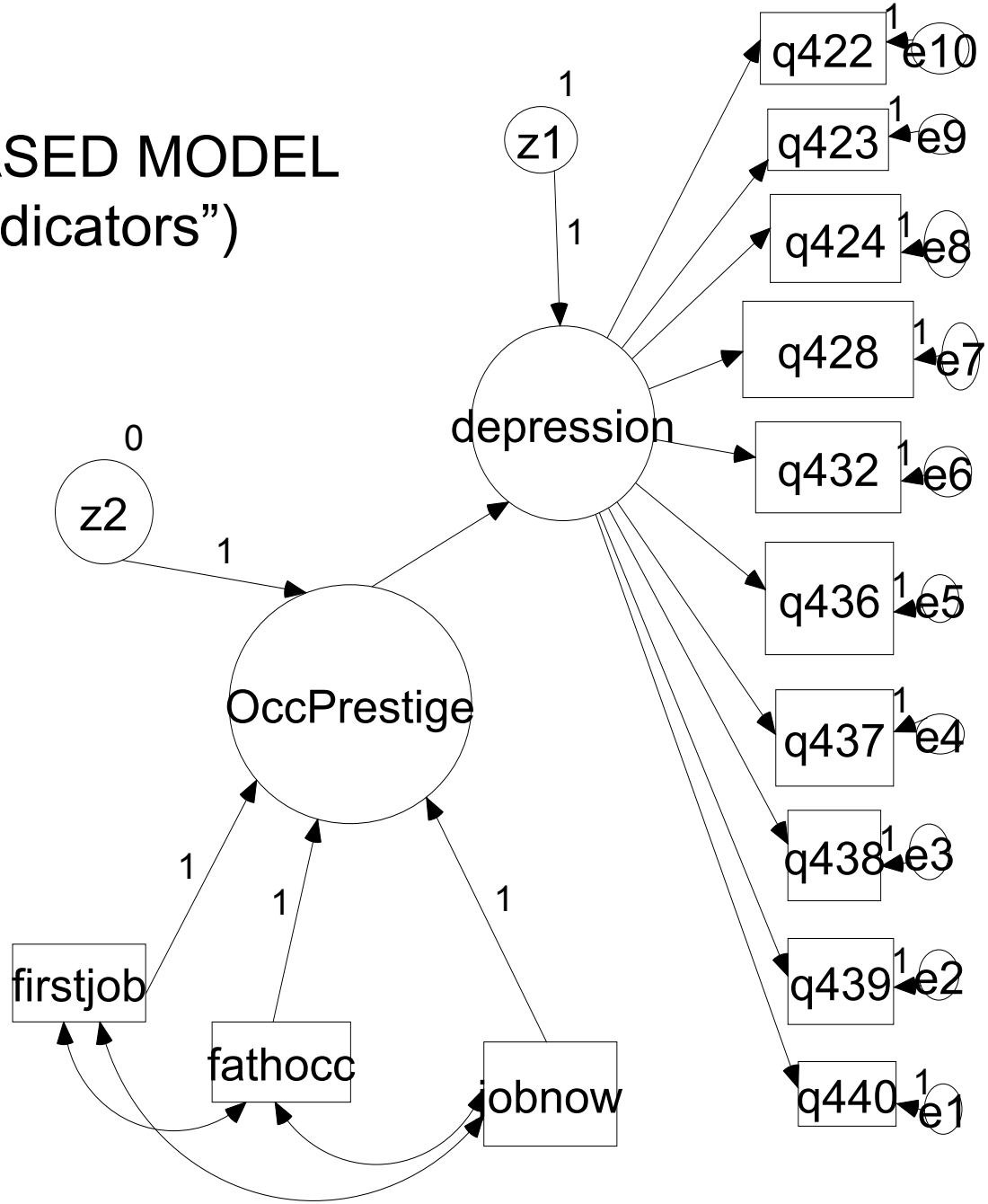
Set variance versus set path coefficient

	Var = 1		Path = 1	
	estimate	Z	estimate	Z
Depr<jobnow	-0.005	-2.3	-0.001	-2.2
Depr<sex	0.039	0.39	0.007	0.38
Depr<age	-0.009	-1.4	-0.002	-1.4
Q422<depr	0.18	7.7	**1.00**	
Q423	0.23	10.2	1.32	6.4
Q424	0.17	6.25	0.96	5.0
Q428	0.14	6.16	0.82	4.9
Q432	0.18	8.13	1.01	5.8
Q436	0.45	15.8	2.57	7.3
Q437	0.41	16.8	2.34	7.4
Q438	0.30	15.5	1.69	7.2
Q439	0.27	15.4	1.53	7.2
Q440	0.41	15.8	2.32	7.3
Chisquare	392.03		392.03	

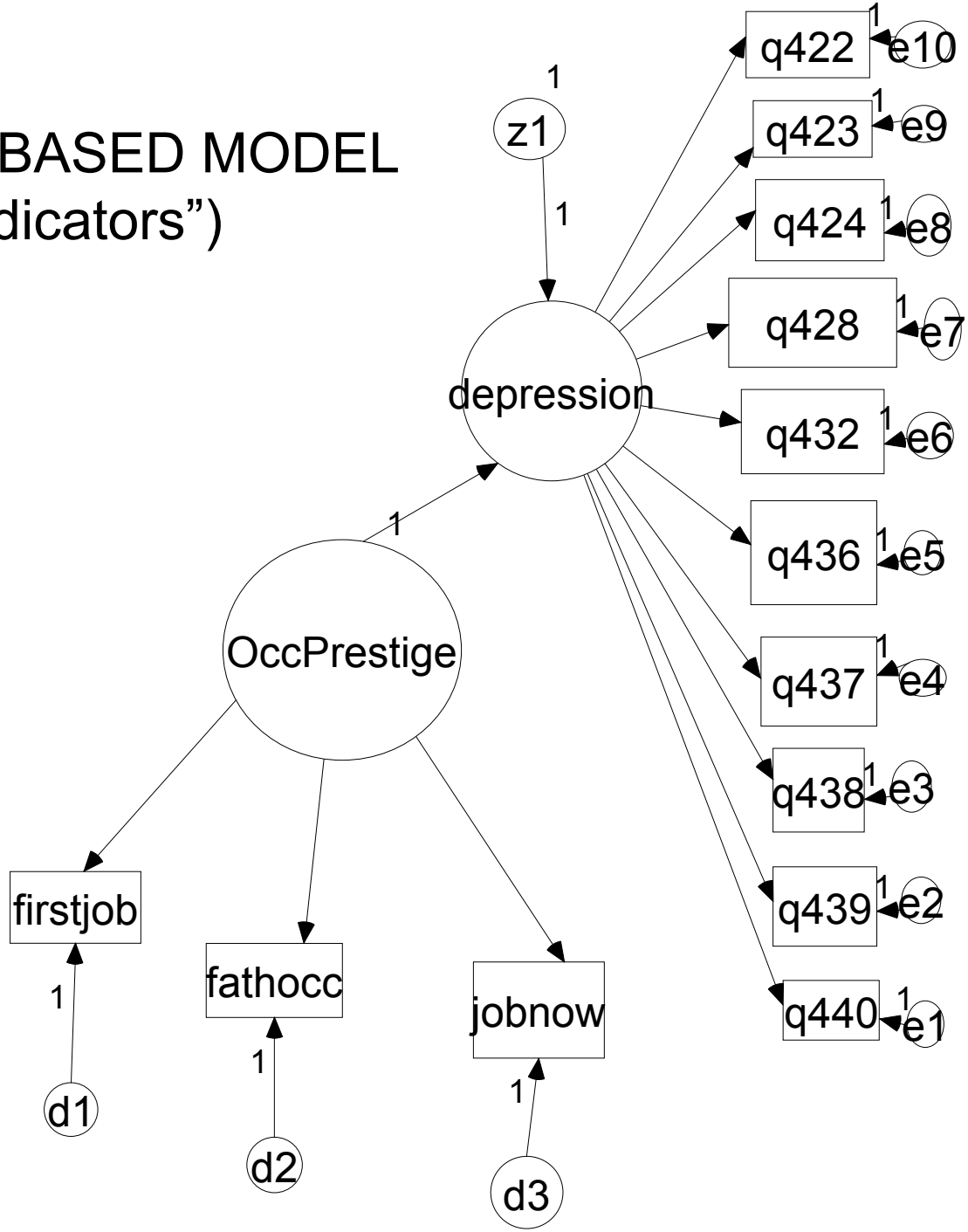
Relating Latent Variables

- What if we want to define a latent variable describing SES or occupational prestige.
- Indicators of job potential:
  - first job
  - current job
  - father's occupation
- Remove age and gender
  - simple
  - insignificant in previous model

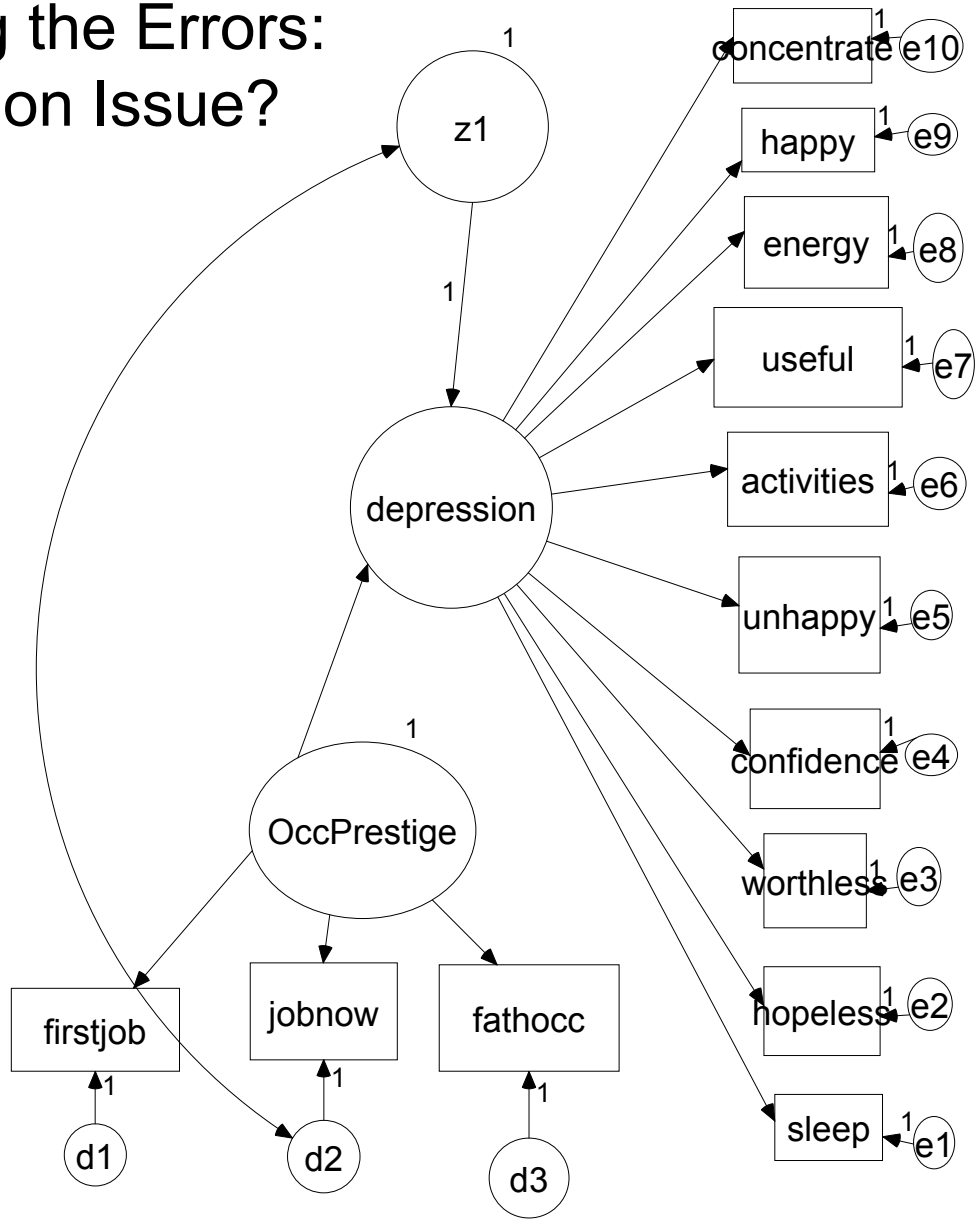
INDEX-BASED MODEL
(“Cause Indicators”)



FACTOR-BASED MODEL
(“Effect Indicators”)



Correlating the Errors: Interpretation Issue?



Equations for model

- Measurement Pieces:

$$firstjob = \lambda_{11}\xi + \delta_1$$
$$fathocc = \lambda_{12}\xi + \delta_2$$
$$jobnow = \lambda_{13}\xi + \delta_3$$

$$y_k = \lambda_k \eta + \varepsilon_k, \quad k = 1,\ldots,10$$

- Structural Piece: $\eta = \gamma\xi + \zeta$

- Correlation of errors: $cor(\zeta_1, \delta_2) \neq 0$

Results

Regression Weights:	Estimate	S.E.	C.R.
depress <--- OccPrestige	0.055	0.069	0.806
q422 <-------- depress	0.178	0.023	7.739
q423 <-------- depress	0.234	0.023	10.251
q424 <-------- depress	0.170	0.027	6.257
q428 <-------- depress	0.146	0.024	6.179
q432 <-------- depress	0.180	0.022	8.144
q436 <-------- depress	0.457	0.029	15.882
q437 <-------- depress	0.416	0.025	16.842
q438 <-------- depress	0.300	0.019	15.438
q439 <-------- depress	0.271	0.018	15.287
q440 <-------- depress	0.413	0.026	15.809
firstjob <-- OccPrestige	16.189	1.924	8.416
fathocc <--- OccPrestige	9.321	1.394	6.687
jobnow <---- OccPrestige	12.488	1.619	7.712

Results

Standardized Regression Weights:	Estimate
depression <------ OccPrestige	0.055
q422 <-------------- depression	0.362
q423 <-------------- depression	0.467
q424 <-------------- depression	0.296
q428 <-------------- depression	0.292
q432 <-------------- depression	0.379
q436 <-------------- depression	0.674
q437 <-------------- depression	0.705
q438 <-------------- depression	0.659
q439 <-------------- depression	0.654
q440 <-------------- depression	0.671
firstjob <-------- OccPrestige	0.673
fathocc <--------- OccPrestige	0.394
jobnow <---------- OccPrestige	0.537

What about gender?

- Gender is a well-known “risk factor” for depression
- And, some research even suggests “differential measurement” for depression by gender
- Is it reasonable to think that SES related variables would be differentially associated with depression by gender?

Categorical Variables

- Assuming normal distribution of variables
  - Structural equation models are based on LINEAR regressions.
  - recall that we use covariance and correlation matrix for estimation
  - Categorical variables don't fit into the interpretation so well, especially when using standardized coefficients.
- One solution: stratify by categories
  - perform separate analyses
  - compare models across groups
  - in AMOS: "group manager"
  - Stronger than adjustment
- Can't always fit one big model that allows for stratification

Stratifying by Gender

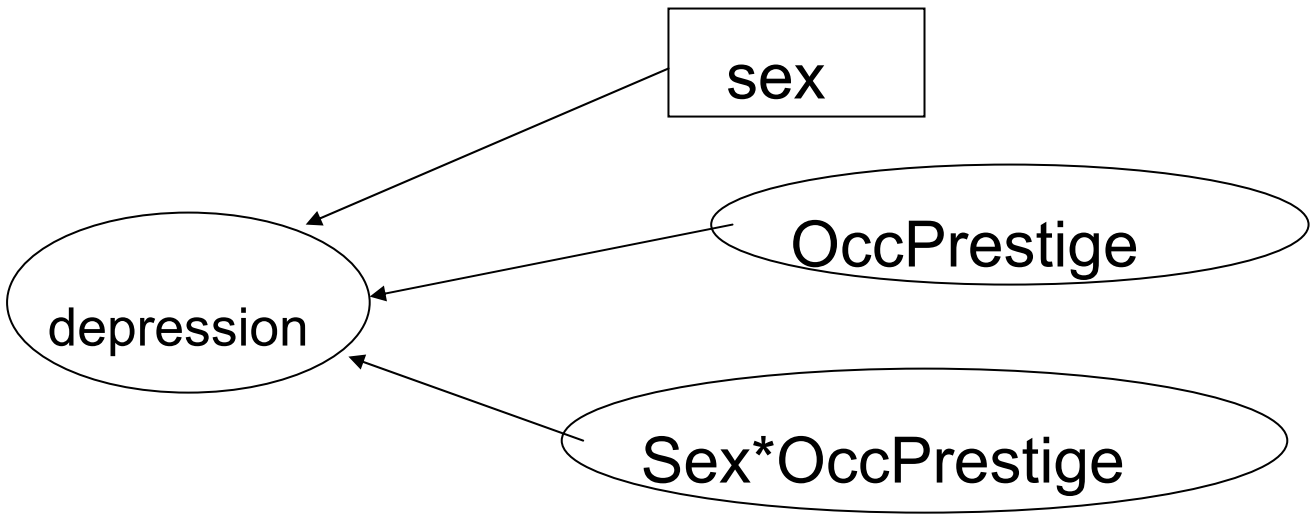
	All	Males	Females
Depr<-OccPrestige	**0.05**	**-0.09**	**0.12**
Q422<-Depr	0.18	0.17	0.19
Q423<-Depr	0.23	0.20	0.26
Q424<-Depr	0.17	0.17	0.16
Q428<-Depr	0.15	0.15	0.13
Q432<-Depr	0.18	0.15	0.21
Q436<-Depr	0.46	0.39	0.50
Q437<-Depr	0.42	0.43	0.39
Q438<-Depr	0.30	0.36	0.25
Q439<-Depr	0.27	0.32	0.23
Q440<-Depr	0.41	0.35	0.47
Firstjob<- OccPrestige	16.2	13.1	19.9
Fathocc<- OccPrestige	9.32	8.10	9.21
Jobnow<- OccPrestige	12.5	14.9	9.99
Correlation between d3 and z1	-0.17*	-0.03	-0.18*

Interpretation of Gamma?

- How do we interpret the association between depression and occupational prestige?
- What does it depend on?
- What approach is easiest?

Testing Gender

- Add gender to model
- Consider results from stratification
  - gender difference in job potential
  - no gender difference in depression
- How to deal with different association between the errors?



Can we do this in our path analysis?

Non-Differential Measurement Assumption

- Recall that our model imposes the following

$$P(y \mid \eta, x) = P(y \mid \eta)$$

$$P(\eta \mid \xi, x) = P(\eta \mid \xi)$$

- Here, x is gender
- Is this true?
- Does knowing gender AND OccPrestige tell us more about depression than just OccPrestige alone?
- Violation of NDM assumption?
  - can't combine gender groups in summarizing association between job potential and depression
  - "Interaction": different relationship between OccPrestige and depression for men and women.

Model Checking via Residuals

- Recall that for a model that fits well, the observed covariance matrix is “similar” to that predicted by model.
- Statistic for checking:

$$\frac{\hat{\text{cov}} - \text{cov}}{se(\hat{\text{cov}})}$$

- Get one for each entry in cov matrix
- Should be between -2 and 2.
- AMOS: residual moments

Why and When SEM?

- When does SEM provide additional benefits?
- Non-recursive models (i.e., no correlated errors or feedback)
  - Can fit separate regressions that overall “define” model
  - Will get same results as if you fit them simultaneously
- Recursive models (correlated errors or feedback)
  - Fitting equations separately gives DIFFERENT results
  - Makes sense: no way to account for correlation of errors

Why and When SEM?

- Caveat 1: just because you assume non-correlated errors doesn't mean it is true!
  - One benefit of using SEM approach is that you can TEST for recursivity
- Caveat 2: model fit and comparisons
  - if you fit separate regressions you have no way to evaluate "total" fit
  - Doesn't allow you to compare overall structures and/or assess fit

Which Estimation Approach to Use?

- Maximum likelihood approach. Estimates are:
  - Asymptotically Unbiased
  - Consistent
  - Asymptotically Efficient
  - Asymptotically Normal
  - PLUS, they are
    - Scale invariant: estimation doesn't depend on whether correlation or covariance matrix was used
    - Scale free: linear transformations of variables will not affect inference.
    - Gives us deviance statistic for comparing models and testing fit

Which Estimation Approach to Use?

- Generalized Least Square approach.
- If the “standard” weights are used (i.e., sample covariance matrix), then we have the following for estimates:
  - Asymptotically Unbiased
  - Consistent
  - Asymptotically Efficient
  - Asymptotically Normal
  - PLUS, they are
    - Scale invariant: estimation doesn’t depend on whether correlation or covariance matrix was used
    - Scale free: linear transformations of variables will not affect inference.
    - Gives us deviance statistic for comparing models and testing fit
  - And one more: doesn’t depend on any normality assumptions.

Which Estimation Approach to Use?

- Unweighted Least Square approach. Estimates are
  - Consistent
- They are NOT
  - Asymptotically Efficient
  - Scale invariant
  - Scale free

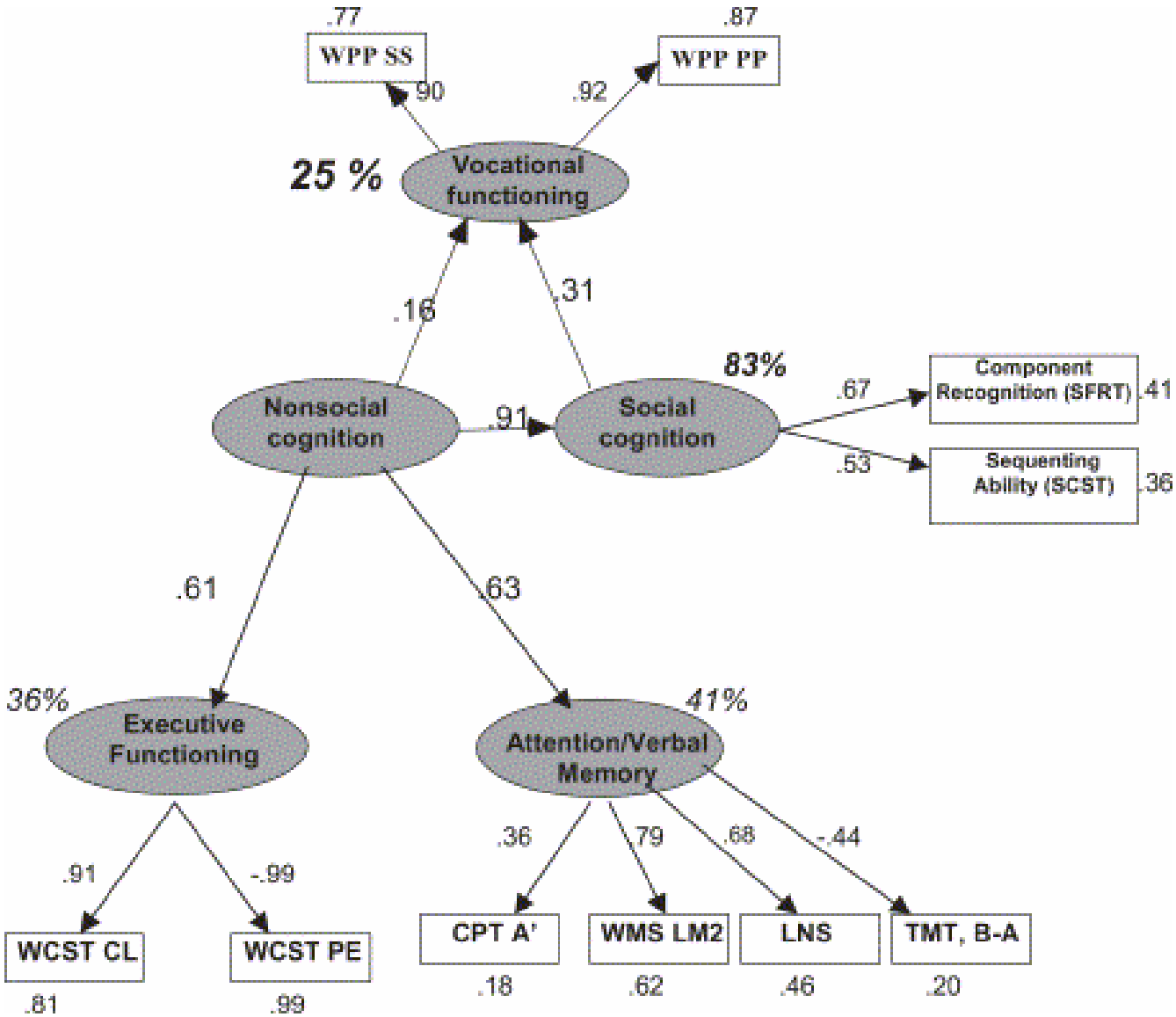
Take Home Points

1. Both GLS and ML are good approaches for estimation
2. GLS may not be very robust in small samples
3. GLS does not depend on normality assumptions, unlike ML
4. ULS will sometimes give you similar results to ML or GLS, but not scale-free or scale-invariant. BAD!

Does social cognition influence the relation between neurocognitive deficits and vocational functioning in schizophrenia?

- **Roland Vauth , Nicolas Rüsch, Markus Wirtz, and Patrick W. Corrigan. Does social cognition influence the relation between neurocognitive deficits and vocational functioning in schizophrenia? Psychiatry Research, Volume 128, Issue 2, 30 September 2004, Pages 155-165**

- Research on barriers to treatment and rehabilitation readiness in people with schizophrenia, especially focusing on risk factors of poor outcome in social and vocational functioning, has focused on the role of social cognition and neurocognition. Others have hypothesized that social cognition (i.e., encoding and understanding of social cues guided by social schemas or scripts) may be one mediator between basic neurocognition and functional outcome. Our study analyzes data from 133 DSM-IV schizophrenic inpatients on a rehabilitation ward using structural equation modeling (SEM) to test whether social cognition has a stronger and more direct influence on vocational functioning than nonsocial cognition. The results supported the hypothesized model; that is, 25% of work-related social skills could be explained by social cognition and nonsocial cognition. The direct impact of nonsocial cognition on vocational functioning was smaller than the impact of social cognition on work-related social skills. Nevertheless, an overwhelming proportion of social cognition (83%) could be explained by nonsocial cognition.

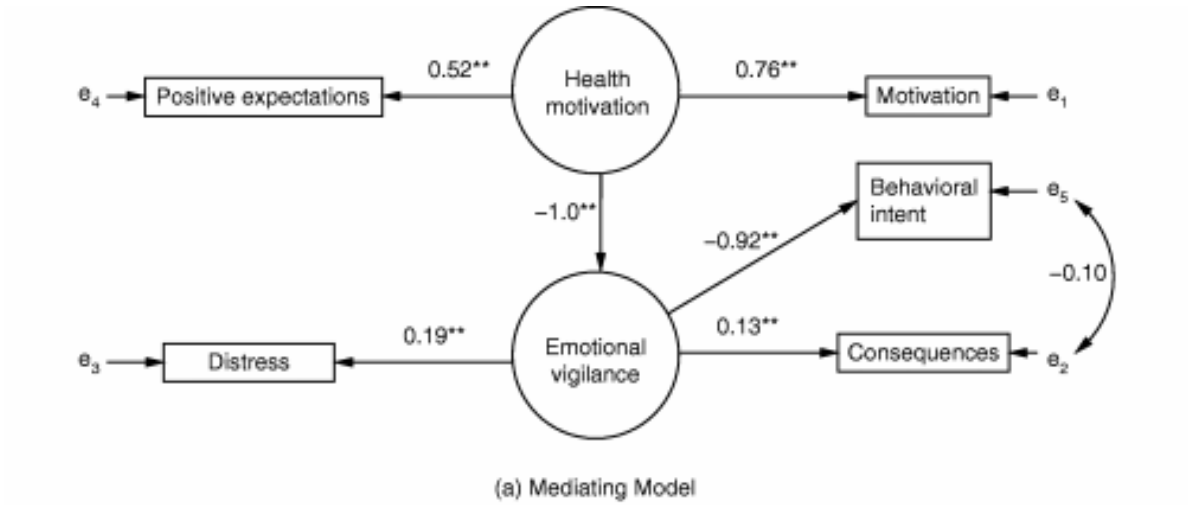

.77
WPP SS
.87
WPP PP
.90
.92
25 %
Vocational functioning
.16
.31
83%
Nonsocial cognition
.91
Social cognition
.67
Component Recognition (SFRT)
.41
.53
Sequenting Ability (SCST)
.36
.61
.63
36%
Executive Functioning
41%
Attention/Verbal Memory
.91
-.99
.36
.79
.68
-.44
WCST CL
WCST PE
CPT A'
WMS LM2
LNS
TMT, B-A
.81
.99
.18
.62
.46
.20

Predicting vocational functioning by social cognition and nonsocial cognition. Empirical data are shown regarding the impact of nonsocial cognition and social cognition on vocational functioning. Structural equation model: rectangles indicate observed indicator variables. Ovals indicate unobserved latent variables. Numbers on single-headed arrows indicate standardized regression weights. Numbers on variables indicate squared multiple correlation coefficients. There were no undefined matrixes and no constrained parameters. The overall model fit was 2=37.7, *df*=30, *P*<0.16. Fit indexes: *C*min/*df*=1.23, NFI=0.99, Tucker–Lewis index (Bentler and Bonnet nonformed fit index)=0.99, RMSEA=0.049. SFRT-2 A′=Situational Feature Recognition Test, second version: A′=sensitivity score, determined from hit and false alarm rates (Corrigan et al., 1996a and Corrigan et al., 1996b); SCST-R=Schema Component Sequencing Task-Revised (Corrigan and Addis, 1995): total number of paired actions correctly juxtaposed to neighboring actions; DS-CPT, A′=Degraded Stimulus Continuous Performance Test, sensitivity score; WCST=Wisconsin Card Sorting Test, perseverative error score and conceptual level answers (Heaton et al., 1993); WMS-R=Wechsler Memory Scale, Revised Version, delayed recall in Logical Memory subtest (Wechsler, 1987); TMT=Trail-making Test, B−A (Reitan and Wolfson, 1995); PANSS=Positive and Negative Syndrome Scale (Kay et al., 1986 and Kay et al., 1987); WPP=Work Personality Profile; Bolton and Roessler, 1986).

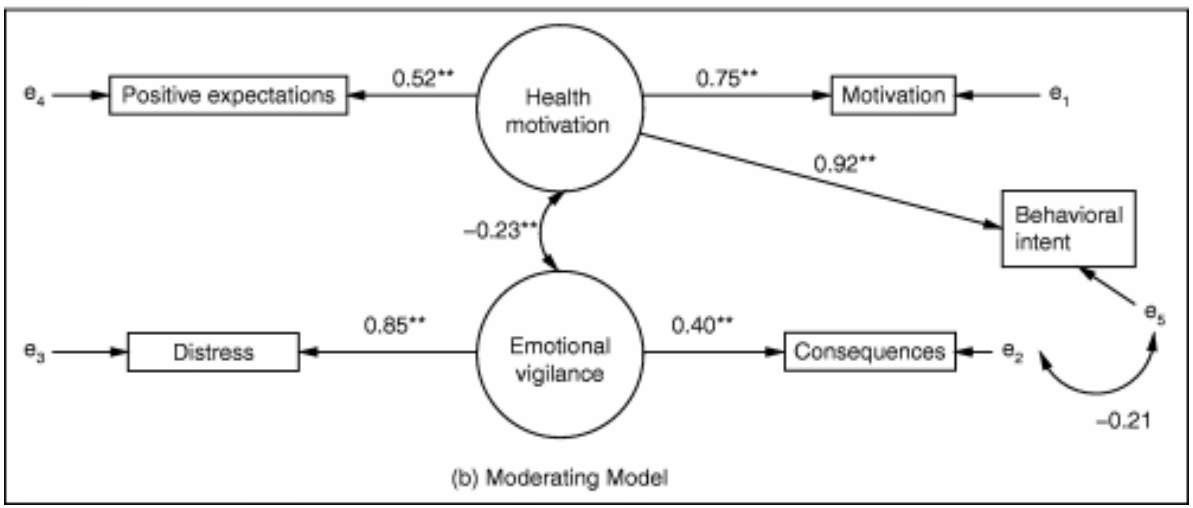
Health motivation and emotional vigilance in genetic testing for prostate cancer risk.

- **Li, Y & Doukas, DJ Health motivation and emotional vigilance in genetic testing for prostate cancer risk. C*linical Genetics* 66 (6), 512-516. December 2004.**
- Actual uptake of genetic testing for cancer susceptibility is generally lower than 50%, despite a high initial interest above 80%. As population-based genetic testing for cancer susceptibility becomes more widespread, there will be an increasing need to understand the relationship of patient-affective factors to test intention and actual uptake behavior. Using hypothetical genetic testing for prostate cancer susceptibility as an example, we used surveys of 400 men in the general population of Philadelphia to develop a Structural Equation Modeling diagram to reveal the influence of affective factors implicated in the intention to undergo genetic testing for prostate cancer risk. Results showed that most men want genetic testing for prostate cancer, believe strongly in its benefits, and are not deterred by negative affect. Our data suggest that high positive expectations, plus a high desire to comply with physician and family suggestions, result in an increased test intention. Informed consent assessment, therefore, requires an appreciation not only of patient risk, but awareness of patient motivation and affect as well.

Fig. 1. Structural Equation Models. Model (a) is the full model. By the conventions of developing an Structural Equation Modeling (SEM) diagram, the four subscales are represented in rectangles and the latent constructs health motivation and emotional vigilance are represented by circles. Standardized estimates of the path coefficients are also plotted and identified with asterisks for statistical significance, with one asterisk indicating $p < 0.001$. Coefficients with no asterisks have $p > 0.05$.



(a) Mediating Model



(b) Moderating Model