Confirmatory Factor Analysis

Session 9, Lecture 7

11/20/06

Adding Latent Variables to the Model

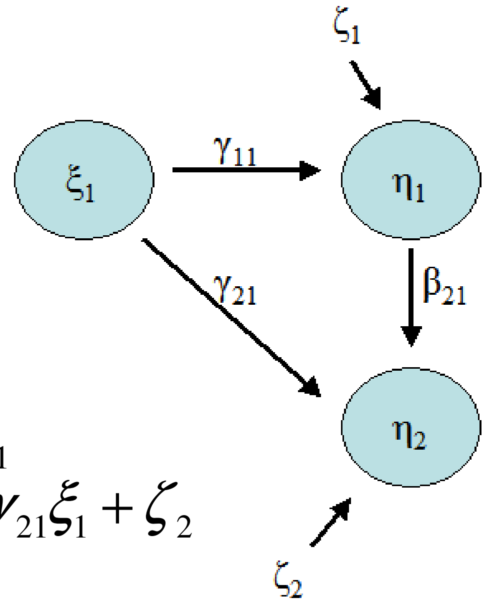
- So far…we've only included **observed** variables in our "path models"
- Extension to latent variables:
  - need to add in a "measurement piece"
  - how do we "define" the latent variable (think factor analysis)
  - more complicated to look at, but same general principles apply

Why we need the "measurement" piece

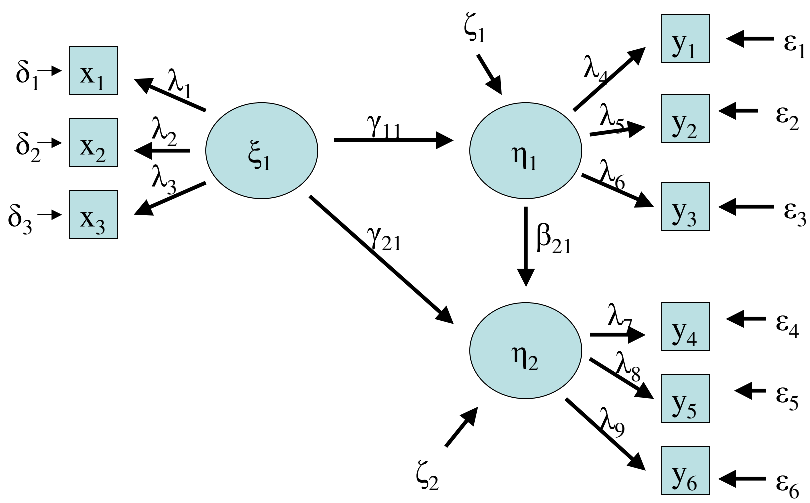
- We can't directly measure most of the interesting things in behavioral science.
- Beyond that, human behavior plays a significant role in most (if not all!) of the pressing public health issues in the US – imagine if we could get every American to
  - Exercise and eat a healthy diet
  - stop smoking, and using drugs or alcohol
  - Use condoms
- This is why we need to figure out how to fit latent variables into a statistical framework

Notation for Latent Variable Model

- η = latent endogenous variable (eta)
- y = observed indicator of η (these both makes "ee" sounds)
- ξ = latent *ex*ogenous variable (pronounced ksi, or *zi*)
- x = observed indicator of ξ (these both make "x" sounds)
- ζ = latent error (zeta) (zeta rhymes with eta)
- β = coefficient on latent endogenous variable (beta)
- γ = coefficient on latent exogenous variable (gamma)
- δ = measurement error on x (delta)
- ε = measurement error on y (epsilon)
- $\Lambda_{y,,} \lambda_y$= coefficient relating y to η (lambda)
- Λ_x, λ_x = coefficient relating x to ξ

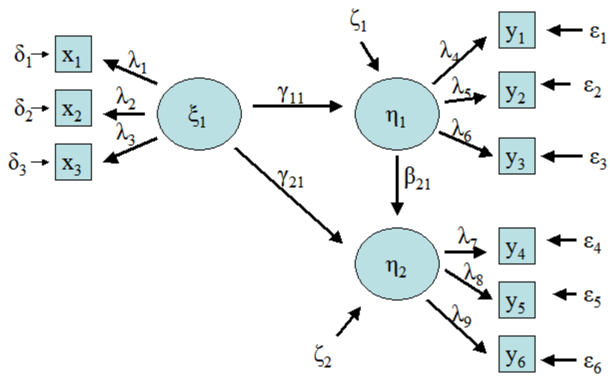


$$\eta_1 = \gamma_{11}\xi_1 + \zeta_1$$
$$\eta_2 = \beta_{21}\eta_1 + \gamma_{21}\xi_1 + \zeta_2$$



To write equations for models:

1. Start an equation for everything that has an arrow pointing at it
2. Each is the sum of (everything pointing at it multiplied by the weight).



$$\eta_1 = \gamma_{11}\xi_1 + \zeta_1$$
$$\eta_2 = \beta_{21}\eta_1 + \gamma_{21}\xi_1 + \zeta_2$$

$$x_1 = \lambda_1\xi_1 + \delta_1 \qquad y_1 = \lambda_4\eta_1 + \varepsilon_1 \qquad y_4 = \lambda_7\eta_2 + \varepsilon_4$$
$$x_2 = \lambda_2\xi_1 + \delta_2 \qquad y_2 = \lambda_5\eta_1 + \varepsilon_2 \qquad y_5 = \lambda_8\eta_2 + \varepsilon_5$$
$$x_3 = \lambda_3\xi_1 + \delta_3 \qquad y_3 = \lambda_6\eta_1 + \varepsilon_3 \qquad y_6 = \lambda_9\eta_2 + \varepsilon_6$$

Confirmatory vs. Exploratory Factor Analysis

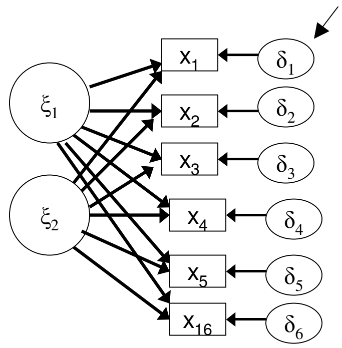
- CFA: We can “test” or “confirm” or “implement” via model constraints.
- Model Construction
  - CFA: drawn in advance, including number of latent variables (factors) and which latent variables influence which indicators.
  - EFA: not in advance, typically all latent variables influence all indicators
- Measurement Errors on Indicators (δ)
  - CFA: may be correlated
  - EFA: can’t be correlated

Exploratory Factor Analysis

Two factor model: $\tilde{x} = \Lambda\tilde{\xi} + \tilde{\delta}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \\ \lambda_{31} & \lambda_{32} \\ \lambda_{41} & \lambda_{42} \\ \lambda_{51} & \lambda_{52} \\ \lambda_{61} & \lambda_{62} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{bmatrix}$$

No curved arrows between deltas



Explanation of Matrix Notation

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \\ \lambda_{31} & \lambda_{32} \\ \lambda_{41} & \lambda_{42} \\ \lambda_{51} & \lambda_{52} \\ \lambda_{61} & \lambda_{62} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{bmatrix}$$

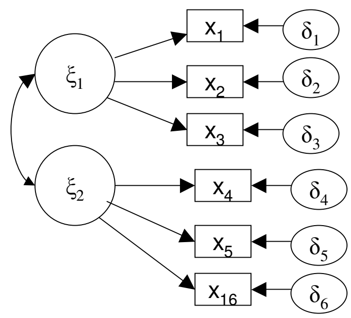
$$x_1 = \begin{bmatrix} \lambda_{11} \\ \lambda_{12} \end{bmatrix} \times \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \delta_1$$

$$= \lambda_{11}\xi_1 + \lambda_{12}\xi_2 + \delta_1$$

Confirmatory Factor Analysis

Two factor model: $\tilde{x} = \Lambda \tilde{\xi} + \tilde{\delta}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \lambda_{11} & 0 \\ \lambda_{21} & 0 \\ \lambda_{31} & 0 \\ 0 & \lambda_{42} \\ 0 & \lambda_{52} \\ 0 & \lambda_{62} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{bmatrix}$$



Don't Like Matrices?

CFA

$$x_1 = \lambda_{11}\xi_1 + \delta_1$$
$$x_2 = \lambda_{21}\xi_1 + \delta_2$$
$$x_3 = \lambda_{31}\xi_1 + \delta_3$$
$$x_4 = \lambda_{42}\xi_2 + \delta_4$$
$$x_5 = \lambda_{52}\xi_2 + \delta_5$$
$$x_6 = \lambda_{62}\xi_2 + \delta_6$$
$$\operatorname{cov}(\xi_1, \xi_2) = \varphi_{12}$$

EFA

$$x_1 = \lambda_{11}\xi_1 + \lambda_{12}\xi_2 + \delta_1$$
$$x_2 = \lambda_{21}\xi_1 + \lambda_{22}\xi_2 + \delta_2$$
$$x_3 = \lambda_{31}\xi_1 + \lambda_{32}\xi_2 + \delta_3$$
$$x_4 = \lambda_{41}\xi_1 + \lambda_{42}\xi_2 + \delta_4$$
$$x_5 = \lambda_{51}\xi_1 + \lambda_{52}\xi_2 + \delta_5$$
$$x_6 = \lambda_{61}\xi_1 + \lambda_{62}\xi_2 + \delta_6$$
$$\operatorname{cov}(\xi_1, \xi_2) = 0$$

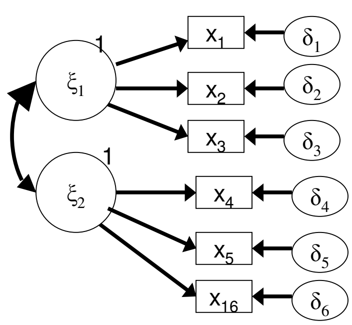
Necessary Constraints

- Latent variables (LVs) need some constraints
- Recall EFA where we constrained factors:

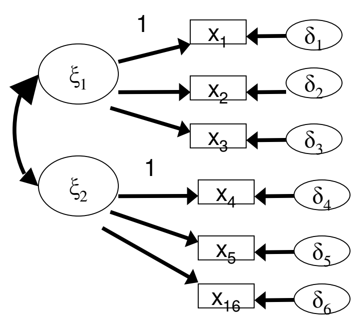
  $F \sim N(0,1)$
- Otherwise, model is not identifiable.

- Here we have two options:
  - Fix variance of latent variables (LV) to be 1 (or another constant)
  - Fix one path between LV and indicator

Fix variances:



Fix path:



Fix variances:

$$x_1 = \lambda_{11}\xi_1 + \delta_1$$
$$x_2 = \lambda_{21}\xi_1 + \delta_2$$
$$x_3 = \lambda_{31}\xi_1 + \delta_3$$
$$x_4 = \lambda_{42}\xi_2 + \delta_4$$
$$x_5 = \lambda_{52}\xi_2 + \delta_5$$
$$x_6 = \lambda_{62}\xi_2 + \delta_6$$
$$\text{cov}(\xi_1, \xi_2) = \varphi_{12}$$
$$\text{var}(\xi_1) = 1$$
$$\text{var}(\xi_2) = 1$$

Fix path:

$$\lambda_{11} = 1$$
$$x_1 = \xi_1 + \delta_1$$
$$x_2 = \lambda_{21}\xi_1 + \delta_2$$
$$x_3 = \lambda_{31}\xi_1 + \delta_3$$
$$x_4 = \xi_2 + \delta_4$$
$$x_5 = \lambda_{52}\xi_2 + \delta_5$$
$$x_6 = \lambda_{62}\xi_2 + \delta_6$$
$$\text{cov}(\xi_1, \xi_2) = \varphi_{12}$$
$$\text{var}(\xi_1) = \varphi_{11}$$
$$\text{var}(\xi_2) = \varphi_{22}$$

More Important Notation

- Φ (capital of φ): covariance matrix of exogenous latent variables

$$\Phi = \text{var}(\xi) = \begin{bmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{12} & \varphi_{22} \end{bmatrix} \qquad \text{var}(\xi_1) = \varphi_{11} \qquad \text{cov}(\xi_1, \xi_2) = \varphi_{12}$$

- Θ (capital of θ): covariance matrix of errors

$$= \text{var}(\delta) = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} & \theta_{14} & \theta_{15} & \theta_{16} \\ \theta_{12} & \theta_{22} & \theta_{23} & \theta_{24} & \theta_{25} & \theta_{26} \\ \theta_{13} & \theta_{23} & \theta_{33} & \theta_{34} & \theta_{35} & \theta_{36} \\ \theta_{14} & \theta_{24} & \theta_{34} & \theta_{44} & \theta_{45} & \theta_{46} \\ \theta_{15} & \theta_{25} & \theta_{35} & \theta_{45} & \theta_{55} & \theta_{56} \\ \theta_{16} & \theta_{26} & \theta_{36} & \theta_{46} & \theta_{56} & \theta_{66} \end{bmatrix}$$

$$\text{var}(\delta_1) = \theta_{11}$$
$$\text{cov}(\delta_1, \delta_2) = \theta_{12}$$

NOTE: usually, $\theta_{ij} = 0$ *if* $i \neq j$

Identifiability Rules for CFA

(1) T-rule (revisited)

- necessary, but not sufficient
- "t" "things" to estimate
- "n" observed variables

$$t \leq \frac{1}{2} n(n+1)$$

2 indicator rule

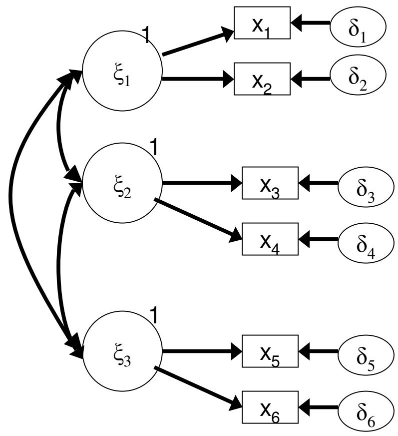
- Sufficient, but not necessary
- At least two factors
- At least two indicators per factor
- Exactly one non-zero element per row of Λ

  (translation: each x is pointed at by one LV)
- Non-correlated errors (Θ_δ is diagonal)

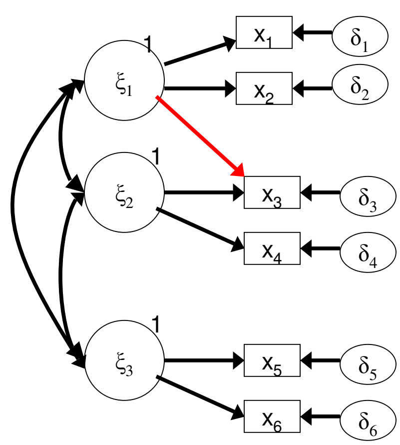
  (translation: no double-header arrows between the δ's)
- Factors are correlated (Φ has no zero elements)*

  (translation: there are double-headed arrows between all of the exogenous latent variables (ξ))

* Alternative less strict criteria: each factor is correlated with **at least** one other factor. (see Bollen, p247)

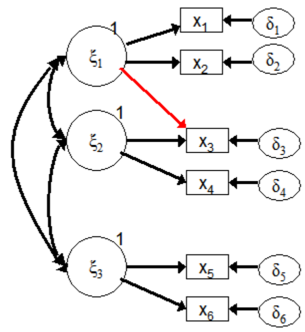


1
1
1
ξ1
ξ2
ξ3
x1
x2
x3
x4
x5
x6
δ1
δ2
δ3
δ4
δ5
δ6



1
1
1
ξ1
ξ2
ξ3
x1
x2
x3
x4
x5
x6
δ1
δ2
δ3
δ4
δ5
δ6

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \lambda_{11} & 0 & 0 \\ \lambda_{21} & 0 & 0 \\ \boxed{\lambda_{31}} & \lambda_{32} & 0 \\ 0 & \lambda_{42} & 0 \\ 0 & 0 & \lambda_{53} \\ 0 & 0 & \lambda_{63} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{bmatrix}$$

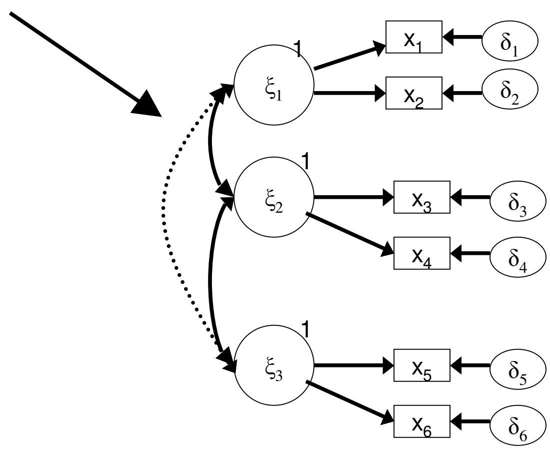


$$\Theta = \mathrm{var}(\delta) = \begin{bmatrix} \theta_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & \theta_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \theta_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta_{66} \end{bmatrix}$$

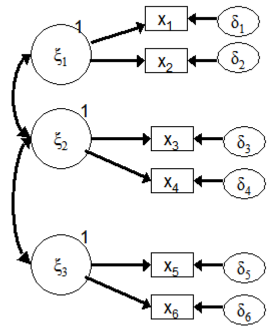
$$\Phi = \mathrm{var}(\xi) = \begin{bmatrix} 1 & \phi_{12} & \phi_{13} \\ \phi_{21} & 1 & \phi_{23} \\ \phi_{31} & \phi_{32} & 1 \end{bmatrix}$$

No zeros in phi =All latent vars are correlated with each other.

What if we take out that arrow?

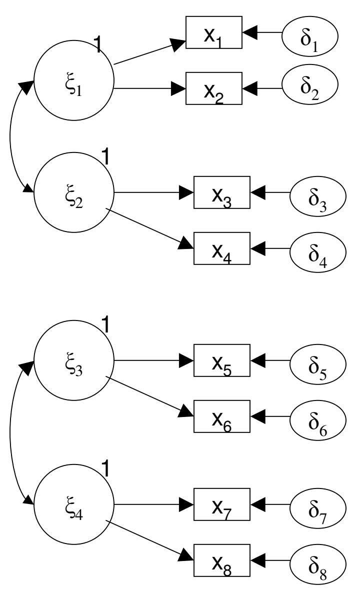


$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \lambda_{11} & 0 & 0 \\ \lambda_{21} & 0 & 0 \\ 0 & \lambda_{32} & 0 \\ 0 & \lambda_{42} & 0 \\ 0 & \lambda_{52} & \lambda_{53} \\ 0 & 0 & \lambda_{63} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{bmatrix}$$

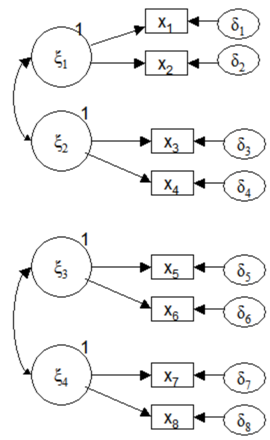


$$\Theta = \text{var}(\delta) = \begin{bmatrix} \theta_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & \theta_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \theta_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta_{66} \end{bmatrix}$$

$$\Phi = \text{var}(\xi) = \begin{bmatrix} 1 & \phi_{12} & \boxed{0} \\ \phi_{21} & 1 & \phi_{23} \\ \boxed{0} & \phi_{32} & 1 \end{bmatrix}$$

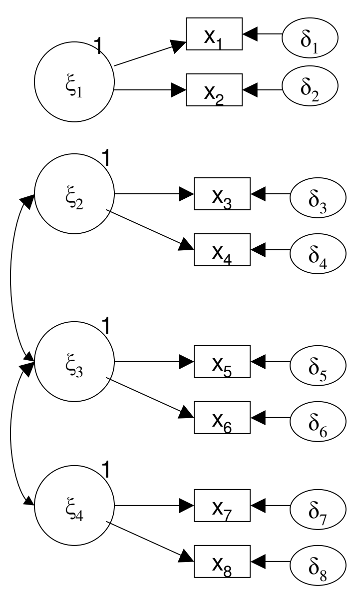


$$
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} =
\begin{bmatrix}
\lambda_{11} & 0 & 0 & 0 \\
\lambda_{21} & 0 & 0 & 0 \\
0 & \lambda_{32} & 0 & 0 \\
0 & \lambda_{42} & 0 & 0 \\
0 & 0 & \lambda_{53} & 0 \\
0 & 0 & \lambda_{63} & 0 \\
0 & 0 & 0 & \lambda_{74} \\
0 & 0 & 0 & \lambda_{84}
\end{bmatrix}
\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{bmatrix} +
\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \end{bmatrix}
$$

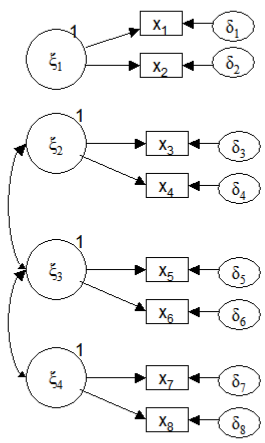


$$
\Theta_\delta =
\begin{bmatrix}
\theta_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \theta_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \theta_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \theta_{11} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \theta_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \theta_{11} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \theta_{11} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{11}
\end{bmatrix}
$$

$$
\Phi =
\begin{bmatrix}
1 & \phi_{12} & 0 & 0 \\
\phi_{12} & 1 & 0 & 0 \\
0 & 0 & 1 & \phi_{34} \\
0 & 0 & \phi_{34} & 1
\end{bmatrix}
$$

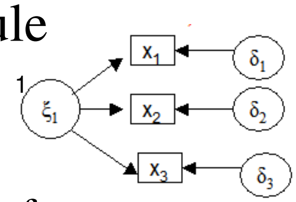


$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} \lambda_{11} & 0 & 0 & 0 \\ \lambda_{21} & 0 & 0 & 0 \\ 0 & \lambda_{32} & 0 & 0 \\ 0 & \lambda_{42} & 0 & 0 \\ 0 & 0 & \lambda_{53} & 0 \\ 0 & 0 & \lambda_{63} & 0 \\ 0 & 0 & 0 & \lambda_{74} \\ 0 & 0 & 0 & \lambda_{84} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \end{bmatrix}$$



$$\Theta_\delta = \begin{bmatrix} \theta_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \theta_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \theta_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \theta_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{11} \end{bmatrix} \qquad \Phi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \varphi_{32} & 0 \\ 0 & \phi_{23} & 1 & \phi_{34} \\ 0 & 0 & \phi_{34} & 1 \end{bmatrix}$$

3 indicator rule



- sufficient, not necessary
- at least one factor
- at least three indicators per factor
- one non-zero element per row of Λ

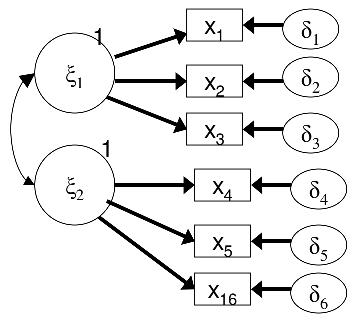
  (translation: each x is pointed at by only one LV)
- non-correlated errors (Θ_δ is diagonal)

  (translation: no double-headed arrows between the δ's)
- NO restrictions on Φ

  (translation: factors don't have to be correlated)

$$\tilde{x} = \Lambda\tilde{\xi} + \tilde{\delta}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \lambda_{11} & 0 \\ \lambda_{21} & 0 \\ \lambda_{31} & 0 \\ 0 & \lambda_{42} \\ 0 & \lambda_{52} \\ 0 & \lambda_{62} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{bmatrix}$$



$$\Theta = \mathrm{var}(\delta) = \begin{bmatrix} \theta_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & \theta_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \theta_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta_{66} \end{bmatrix}$$

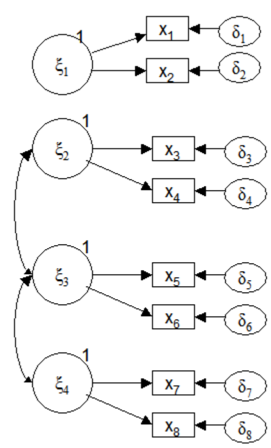
$$\Phi = \mathrm{var}(\xi) = \begin{bmatrix} 1 & \varphi_{12} \\ \varphi_{12} & 1 \end{bmatrix}$$

What about the T-rule?

Sample Moments: (8*9)/2=36

Parameters being estimated: 18

- •exogenous variances=0 (fixed)
- •error variances = 8
- •direct effects = 8
- •double-headed arrows = 2



Food for Thought…

- If something fulfills the 3-indicator rule, will it automatically fulfill the 2-indicator rule?
  - If not, draw a counter-example
- If something fulfills the 2-indicator rule, will it automatically fulfill the 3-indicator rule?
  - If not, draw a counter example
- If something fulfills either the 3- or 2-indicator rule, will it automatically fulfull the t-rule?
  - If not, draw a counter-example

Estimation Procedures

- (Bollen, Ch. 2 and Ch.7)
  - Maximum Likelihood
  - Generalized Least Squares
- Where these are in AMOS:
  - View/Set
  - Analysis Properties
  - Estimation

CMIN (a global test of fit)

- Global Tests ~ Goodness of Fit
- AMOS: CMIN
- Get 1 value comparing the covariance predicted by the model to the observed covariance (v. ugly formula found in AMOS help appendix B)
- The values can be used to compare nested models (e.g., your models compared to the "saturated model")

More on CMIN

- "CMIN": Depends on estimation procedure
- Asymptotically χ^2
  - Maximum likelihood: -2Log-likelihood
  - Generalized LS: like Pearson χ^2
- CMIN for ML and from LS will converge as sample size approaches infinity.
- Assumes "perfect fit": small deviations from model can lead to rejections (especially as N gets large)

Other fit statistics:

- Information Criteria
  - Akaike:

    AIC = -2LL + 2*s
  - Schwarz:

    BIC = -2LL + s*log(N*p)
  - “consistent” AIC:

    CAIC = -2LL + s*(log(N) + 1)

  - s is # of free parameters (parameters being estimated)
  - p is number of parameters in “independence” model (no arrows, no latent vars, just observed variables)
  - the smaller the better

Other fit statistics:

- RMR (root mean square residual)
  - average squared amount that sample variances and covariances differ from estimates
  - 0 is perfect fit
  - the smaller the better, but no preset cutoff

- GFI (goodness of fit index)
  - 1 is perfect fit
  - ranges from 0 to 1
  - According to Garson, you want to see a GFI>.9

- For more, check out http://www2.chass.ncsu.edu/garson/pa765/structur.htm

Comparing Models

- AMOS:
  - Model Fit
  - Manage Models
- Can constrain parameters
- Careful with errors--can get negative variance estimate warnings.
- Nested likelihood ratios
- Comparison of info criteria (AIC, BIC)

Parameter Evaluation

- Estimate/SE = Z-statistic
- Standard interpretation:
  - if $|Z| > 2$, then "significant"
- Consider both statistical and scientific value of including a variable in the model
- Notes for parameter testing in CFA:
  - Not usually interesting in determining if loadings are equal to zero
  - Might be interested in testing whether or not covariance between factors is zero.

CFA Example: Epidemiologic Catchment Area (ECA) Data

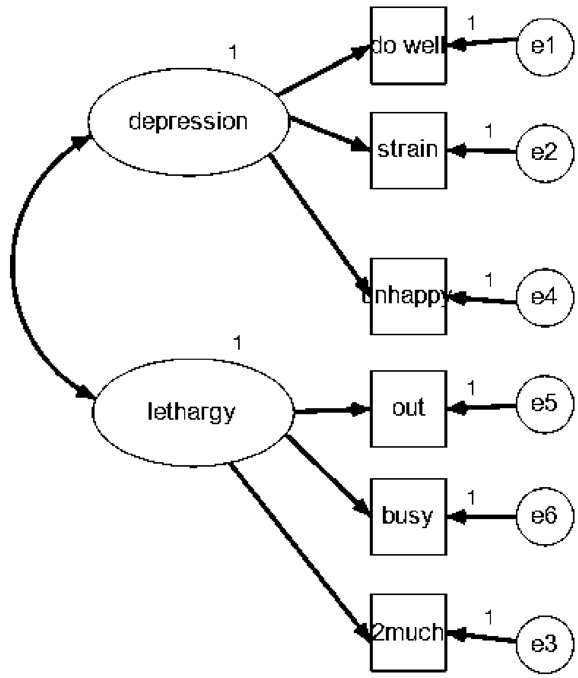
Hypotheses:

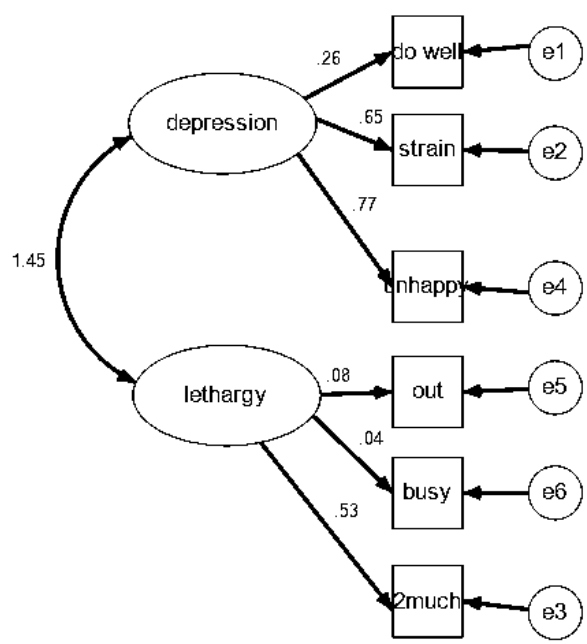
- Lethargy and depression are two aspects of "mental distress."
- Gender and age influence depression
- Current job influences both depression and lethargy

CFA Example: ECA data

- Factors:
  - depression
  - lethargy
- X's:
  - "have you been managing to keep yourself busy and occupied?"
  - "have you been getting out of the house as much as usual?"
  - "have you felt on the whole that you were doing things well?"
  - "have you felt constantly under strain?"
  - "have you found everything getting too much for you?"
  - "have you been feeling unhappy and depressed?"

Is this model identified?
-3 indicator rule
-How many sample moments to I have?
-How many parameters am I estimating?





Everything seems fine, right?

Computation of degrees of freedom (Correlated Factors)

Number of distinct sample moments:	21
Number of distinct parameters to be estimated:	13
Degrees of freedom (21 - 13):	8

Or not…..

Notes for Model (Group number 1 - Correlated Factors)

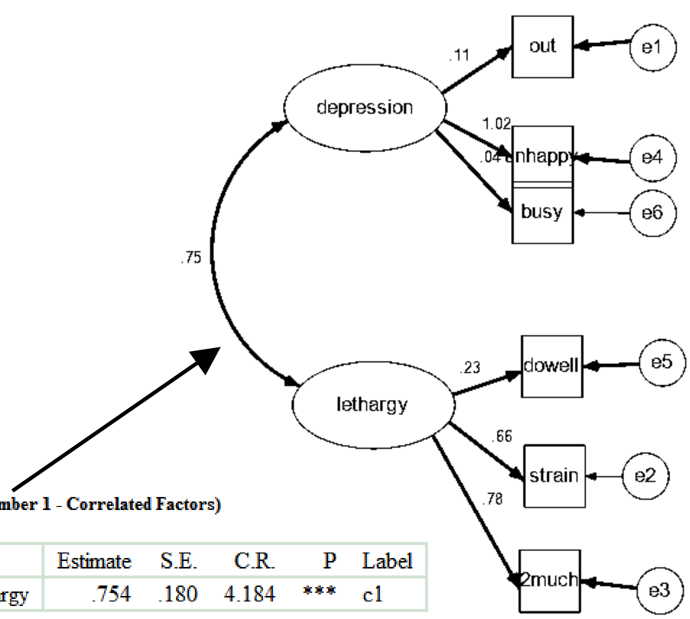
The following covariance matrix is not positive definite (

	lethargy	depress
lethargy	1.000	
depress	1.452	1.000

Is this an identifiability issue?

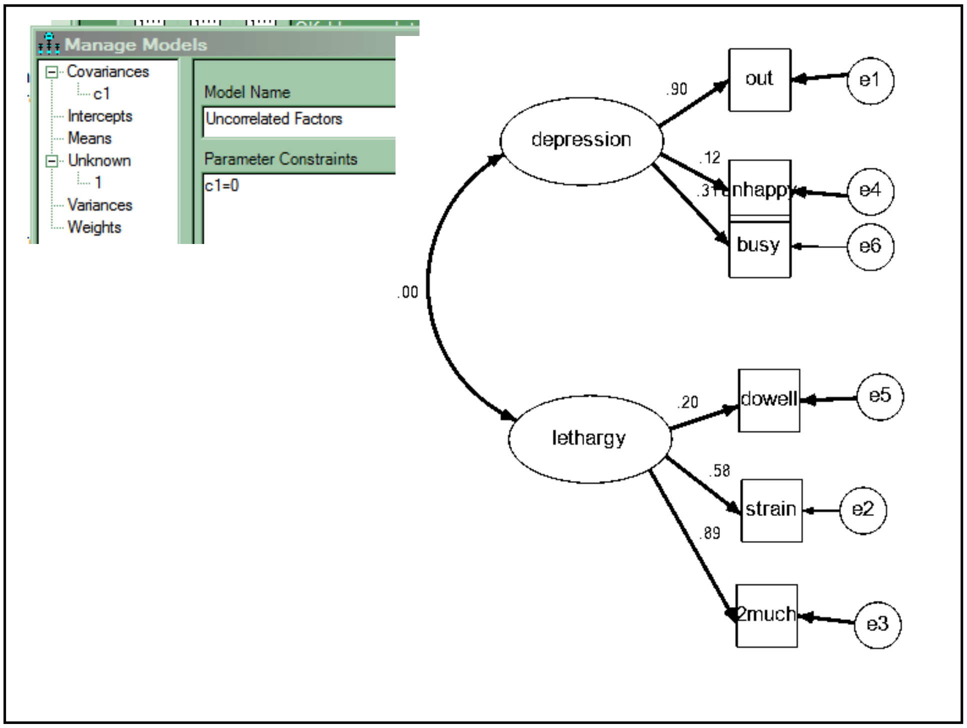
Notes for Group/Model (Group number 1 -

This solution is not admissible.



Covariances: (Group number 1 - Correlated Factors)

	Estimate	S.E.	C.R.	P	Label
depress <--> lethargy	.754	.180	4.184	***	c1



CMIN

Model	NPAR	CMIN	DF	P	CMIN/DF
Correlated Factors	13	134.984	8	.000	16.873
Uncorrelated Factors	12	367.554	9	.000	40.839
Saturated model	21	.000	0		
Independence model	6	593.232	15	.000	39.549

RMR, GFI

Model	RMR	GFI	AGFI	PGFI
Correlated Factors	.035	.912	.768	.347
Uncorrelated Factors	.082	.838	.622	.359
Saturated model	.000	1.000		
Independence model	.100	.703	.585	.502

CMIN= -2LL (for ML)

Proof: AIC = -2LL + 2*s

-2LL= 160.984-(2*13)

= 134.984

AIC

Model	AIC	BCC	BIC	CAIC
Correlated Factors	160.984	161.344	216.133	229.133
Uncorrelated Factors	391.554	391.886	442.461	454.461
Saturated model	42.000	42.581	131.087	152.087
Independence model	605.232	605.398	630.685	636.685

Second-Order Factor Analysis

- Factors:
  - depression
  - lethargy
- X's:
  - "have you been managing to keep yourself busy and occupied?"
  - "have you been getting out of the house as much as usual?"
  - "have you felt on the whole that you were doing things well?"
  - "have you felt constantly under strain?"
  - "have you found everything getting too much for you?"
  - "have you been feeling unhappy and depressed?"

Maybe you look back at this, and think, maybe those two factors are correlated, because there is a "grand" factor to which they are both related.

You would want STRONG prior theory…

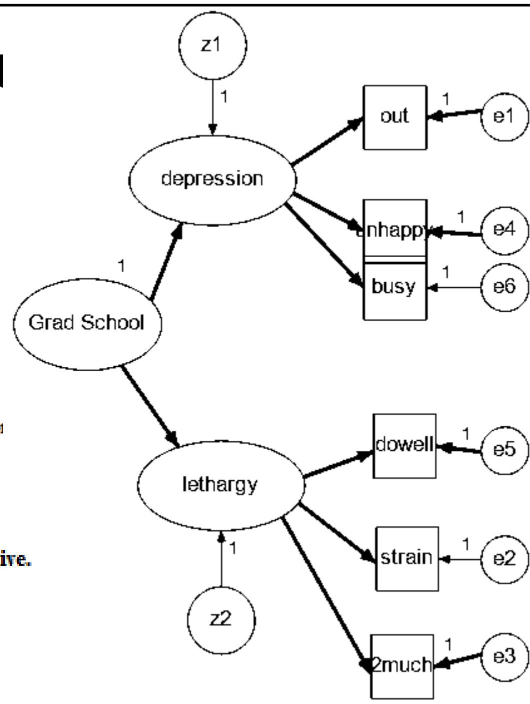
Like mine…

Result (Second Order)

Iteration limit reached
The results that follow are therefore inco
Chi-square = 433.419
Degrees of freedom = 8
Probability level = .000

The following variances are negative.

	z1	e5
	-1.019	-.776



In order to get this to run, I had to keep constraining more and more parameters to be equal to 1.

And still, even if it ran, bad things kept happening…

Regression Weights: (Group number 1 - Second Order)

			Estimate	S.E.
depress	<---	Grad School	.000	419255984600352.000
lethargy	<---	Grad School	.000	319694460478925.000
q426	<---	depress	1.000	
q436	<---	depress	1.000	

Totally crazy 419 trillion! (variance would be 177 octrillions)

But, something even MORE important:
The names of the two mediating latent vars can be (sort of) inferred from its indicators. BUT, what about the name for the exogenous latent variable – It's hard to figure out what things really are when you have latent variables (with no indicators of their own) pointing to other latent variables.