# Causal Inference

Statistics for Psychosocial Research II

Structural Models

November 8, 2006

# Many views on this topic!

- Correlation ≠ Causation
- But, coupled with other information, correlation can imply causation
- Statistics helps a lot with causal inference
- Statistical models used to draw inferences are distinctly different from those used for showing associational differences

# ‘Potential’ Cause

- Holland (1986): each ‘unit’ of observation must be able to be ‘exposed’ to the cause
- For causal inference, cause must be subject to “human manipulation.”
- Does
  - Smoking cause lung cancer?
  - Sleet or snow cause traffic accidents?
  - A change in interest rates cause the stock market to fluctuate?
  - Gender or race cause discrimination?

# Three examples from Holland

(A) She did well on the exam because she is a woman.

(B) She did well on the exam because she was coached by her teacher.

(C) She did well on the exam because she studied for it.

In (A), is there a 'cause'? NO

In (B), is there a 'cause'? YES

In (C), is there a 'cause'? ?

Why (C)? Studying is voluntary: Can we MAKE someone study? We COULD prevent someone from studying. Debatable….

# “Potentially Exposable”

- Every ‘unit’ should be able to be exposed to the cause.
- Good example:  randomized clinical trial
- Need to be able to postulate that we could state what WOULD have happened to a patient’s outcome had the cause been “the reverse”.
  - Assume $Y_{ti}$ is the outcome of $Y_i$ if patient $Y_i$ is in the treatment group
  - Assume $Y_{ci}$ is the outcome of $Y_i$ if patient $Y_i$ is in the control group
- We are interested in the causal effect: $Y_{ti} - Y_{ci}$

# Fundamental Problem of Causal Inference

"It is impossible to *observe* the value of $Y_{ti}$ and $Y_{ci}$ on the same patient. Therefore it is impossible to observe the causal effect of treatment on patient $Y_i$."

- Important point: 'observe' is key word.
- But, we can make inferences using statistical reasoning
- Possible exceptions: cross-over designs in some settings.

# Statistical Solution

- We use information on a number of different patients to gain knowledge about causal effect.
- We cannot estimate causal effects for individuals
- We CAN estimate ‘average’ causal effects over a population of patients.

# Special Cases of Causal Inference

1. Temporal Stability: response does not depend on when exposure occurs.
2. Causal Translucence: prior exposure to cause does not affect outcome.
3. Unit homogeneity: effect is the same when cause is applied to identical units
4. Independence…..This is most relevant to our class…..

# 4. Independence

- Randomized trial is a special (very special!) case of independence.
- We cannot always assume that “exposure” is assigned randomly
- Example:  smoking and health outcomes
  - Is it reasonable to assume that smoking is randomly assigned?
  - What would be the health outcomes of smokers if, holding all else constant, they were instead non-smokers?
  - Would the health outcomes be the same as the ‘true’ non-smokers?
- ASSUMPTION:  The determination of the cause is independent of all other variables, including the outcome of interest.
- Reasonable???

# Causality

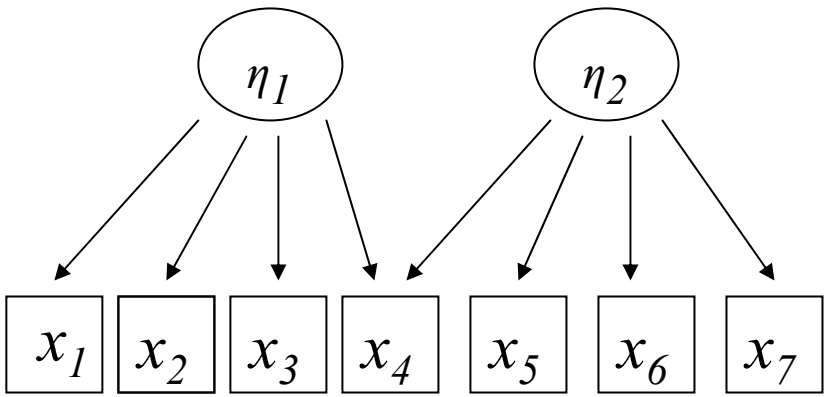
- Strong assumption of causality in SE models
- Does that mean we cannot include ‘gender’ in our models?
- No:  it means that we cannot make ‘causal inferences’ about gender in our models.
- Bollen’s three components for ‘practically’ defining a causal relationship:
  - isolation
  - association
  - direction of influence

# ASSOCIATION

- Easier to establish
- Causal variable should have strong association with outcome
- Problems:
  - incorrect standard errors or test statistics (e.g. correlated errors, poor measures)
  - multicollinearity
- Replication/Repetition important (and also helps establish isolation)

# Multicollinearity Example

$\eta_1$: morale; $\eta_2$: sense of belonging




$$x_1 = \gamma_{11}\eta_1 + \zeta_1$$
$$\vdots$$
$$x_4 = \gamma_{14}\eta_1 + \gamma_{24}\eta_2 + \zeta_4$$
$$\vdots$$
$$x_7 = \gamma_{27}\eta_2 + \zeta_7$$

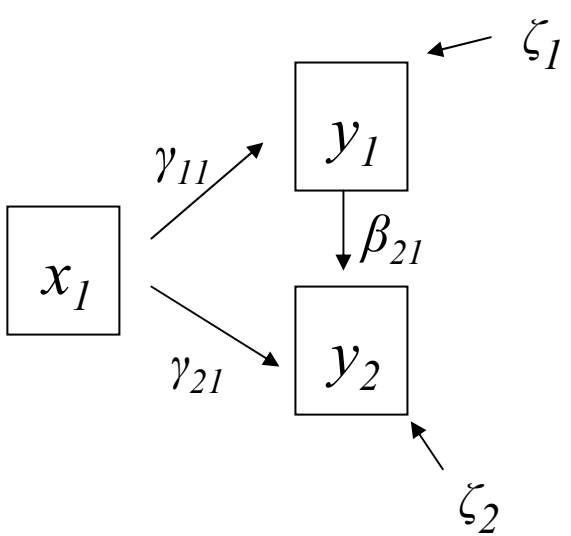
In truth, $x_4$ is a measure of morale, but we allow it to be related to sense of belonging.
Results? Both $\gamma_{14}$ and $\gamma_{24}$ are insignificant.
Why? Because morale and sense of belonging are highly associated.

# Direction of Causation

- Plausibility of association being causal rests on having causal direction correct
- Temporal?
  - x *should* come before y in time
  - problematic:  simultaneous reciprocal causation (feedback) is not possible
  - window of cause and response time
- We often have cross-sectional data.
- Can future event predict past or present event?

# Aside: Total, Direct, and Indirect Effects

- $x_1$ is marital status, $y_1$ is income, $y_2$ is depression
- Direct effect: measured by a single arrow between two variables
- Indirect effects: measured by all possible "paths" or "connections" between two variables EXCEPT for the direct path. We multiply the coefficients on path together to get each indirect effect.
- Total effect: the sum of the direct and indirect paths between two variables




Direct effect of $x_1$ on $y_2$:

Indirect effect(s) of $x_1$ on $y_2$:

Total effect of $x_1$ on $y_2$:

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Direct effect of $y_1$ on $y_2$:

Indirect effect(s) of $y_1$ on $y_2$:

Total effect of $y_1$ on $y_2$:

# ISOLATION

- Isolation:  hold everything constant except the cause and the outcome
- Impossible to establish unless x and y occur in a “vacuum”
- Especially difficult in observational studies!
- Without true isolation can never be 100% certain about cause
- Is that ‘weird’ in statistics?  NO!  We are never 100% certain in statistics!
- **Isolation tends to be the weakest link in determining causality**

# “Pseudo-isolation”

$$y_1 = \gamma_{11} x_1 + \zeta_1$$

- $\zeta_1$ is the unobserved error/disturbance
- $\zeta_1$ represents ALL other causes/correlates of $y_1$
- Standard assumption for pseudo-isolation:

$$\text{Cov}(x_1, \zeta_1) = 0$$

- That is, $x_1$ is independent of all other causes/correlates of $y_1$
- If the assumption is true, then we can assess causal association of $x_1$ and $y_1$ “isolated” from all other causes ($\zeta_1$).

# “Pseudo-isolation”

- Can think of pseudo-isolation as a probabilistic view of causality
- Predictability of $y_1$ lies between two models:

$$y_1 = \gamma_{11} x_1 \quad \leftarrow \quad \text{All Cause}$$

$$y_1 = \zeta_1 \quad \leftarrow \quad \text{All Error}$$

# Practically Speaking

- Unrealistic to think that $x_1$ is the only cause of $y_1$ and that $Cov(x_1, \zeta_1) = 0$.
- We need to account for other factors (e.g. cancer, smoking, coffee example).

$$y_1 = \gamma_{11} x_1 + \gamma_{12} x_2 + \cdots + \gamma_{1m} x_m + \zeta_1$$

- Latent variable approach? Same…..

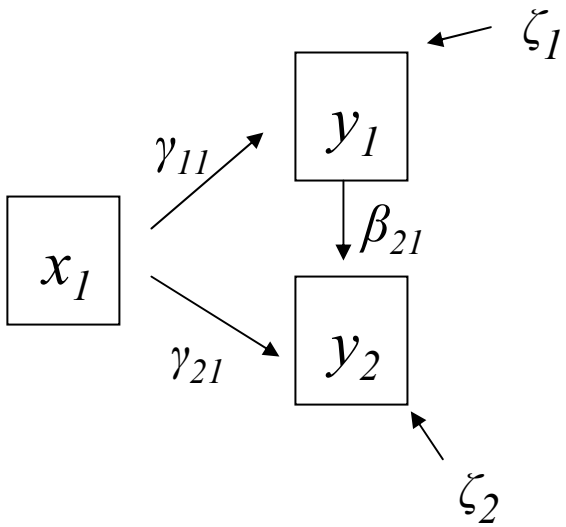
$$y_1 = \gamma_{11} \eta_1 + \gamma_{12} \eta_2 + \cdots + \gamma_{1m} \eta_m + \zeta_1$$

$$Cov(\eta_i, \zeta_1) = 0$$

# Examples of Violations of Isolation

## (1) INTERVENING VARIABLES

True Model:




$$y_1 = \gamma_{11} x_1 + \zeta_1$$

$$y_2 = \beta_{21} y_1 + \gamma_{21} x_1 + \zeta_2$$

$$Cov(\zeta_1, \zeta_2) = 0$$

$$Cov(x_1, \zeta_1) = 0$$

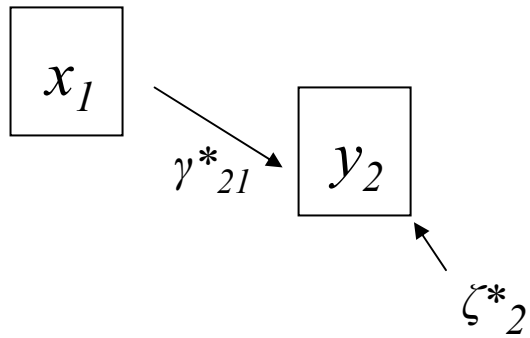
$$Cov(x_1, \zeta_2) = 0$$

(e.g. $x_1$ is marital status, $y_1$ is household income, $y_2$ is depression)

# What if we omit $y_1$ (income)?

- Assumed model:

$$y_2 = \gamma^*_{21} x_1 + \zeta^*_2$$




- This implies:

$$\zeta^*_2 = \beta_{21} y_1 + \zeta_2$$

- And our pseudo-isolation assumption….

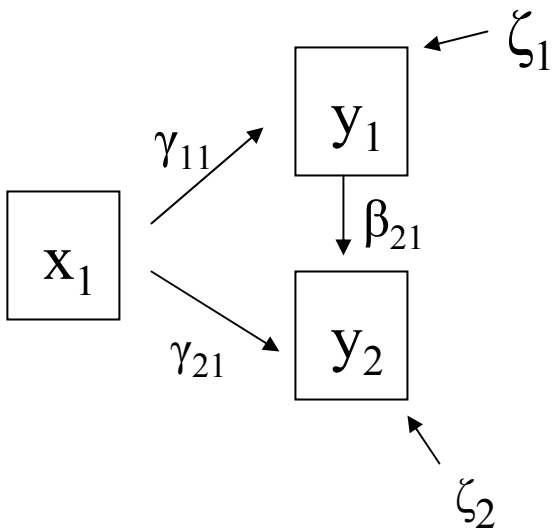
$$\begin{aligned} Cov(x_1, \zeta^*_2) &= Cov(x_1, \beta_{21} y_1 + \zeta_2) \\ &= Cov(x_1, \beta_{21}(\gamma_{11} x_1 + \zeta_1) + \zeta_2) \\ &= \beta_{21}\gamma_{11} Var(x_1) + \beta_{21} Cov(x_1, \zeta_1) + Cov(x_1, \zeta_2) \\ &= \beta_{21}\gamma_{11} Var(x_1) \\ &\neq 0 \end{aligned}$$

# Effect on Inference?

- $\gamma_{21}^{*}$ converges to total effect, $\beta_{21}\gamma_{11} + \gamma_{21}$, instead of direct effect, $\gamma_{21}$
- This yields an over or under-estimate of the effect of $x_1$ on $y_2$.
- Can be a really big problem if direct and indirect effects cancel each other out.
  - If $\gamma_{21} = 1$; $\beta_{21} = 0.5$; $\gamma_{11} = -2$
  - Then, $\gamma_{21}^{*} = -0.5*2 + 1 = 0$
  - We might conclude that there is NO association!

# (2) LEFT OUT COMMON CAUSE

## Recall True Model




$$y_1 = \gamma_{11} x_1 + \zeta_1$$

$$y_2 = \beta_{21} y_1 + \gamma_{21} x_1 + \zeta_2$$

$$Cov(\zeta_1, \zeta_2) = 0$$

$$Cov(x_1, \zeta_1) = 0$$

$$Cov(x_1, \zeta_2) = 0$$

## What if we omit $x_1$ from the model?

$$\text{Then } y_2 = \beta^*_{21} y_1 + \zeta^*_2$$

$$\text{where } \zeta^*_2 = \gamma_{21} x_1 + \zeta_1$$

- Is pseudo-isolation assumption violated?

$$\begin{aligned} Cov(y_1, \zeta_2^*) &= Cov(\gamma_{11} x_1 + \zeta_1, \gamma_{21} x_1 + \zeta_2) \\ &= \gamma_{11}\gamma_{21} Var(x_1) \\ &\neq 0 \end{aligned}$$

- What happens to our estimate of $\beta_{21}$ ?

$$\beta_{21}^* = \beta_{21} + \gamma_{21}\gamma_{11}$$

(again, we get total effect instead of direct)

# Effects on Inference

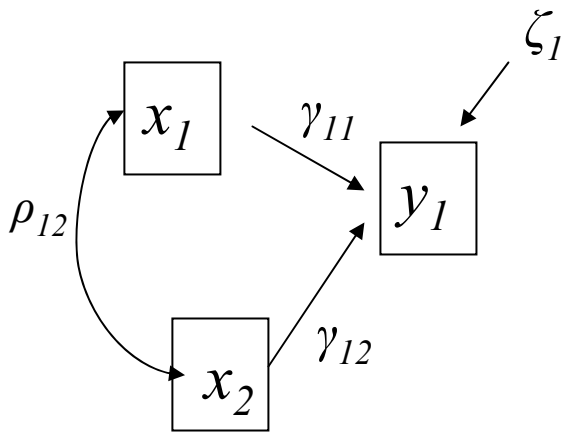
- Worst case scenario: $y_1$ and $y_2$ have little or no association, but both are highly associated with $x_1$.
- Example:

  $x_1$ = age

  $y_1$ = proportion of gray hairs

  $y_2$ = quality of vision
- "Spurious Association"

# (3) OMITTED VARIABLE HAS UNSPECIFIED RELATION TO OTHER VARIABLES




True Model:

$$y_1 = \gamma_{11} x_1 + \gamma_{12} x_2 + \zeta_1$$

- What if we omit $x_2$?
  - Assumed model is $y_1 = \gamma^*_{11} x_1 + \zeta^*_1$
  - And $\gamma^*_{11} = \gamma_{11} + \rho_{12}\gamma_{12}$

# This is an even bigger problem….

- Note that the association between $x_1$ and $x_2$ is unspecified: It could be true that
  - $x_1$ causes $x_2$ and $y_1$ (intervening variable)
  - $x_2$ causes $x_1$ and $y_1$ (common cause)
  - something else
- We can't infer about the exact consequences of omitting $x_2$ because we don't know its association to the other variables.

# Other Violations

- “feedback” or “reciprocal causation”
- Wrong functional form between 2 variables
- Correlated errors