Sample Size Considerations in Factor Analysis and Latent Class Analysis

October 18, 2006

Statistics for Psychosocial Research

Outline of Today's Discussion

- Sample Size for reliability analyses
  - Confidence intervals for reliability
  - Hypothesis testing in the context of reliability
- Sample Size in factor analysis
- Sample size in latent class analysis

Statistical Power

- Power describes the ability to detect associations if they truly exist
- Determined by
  - Difference desired to be detected
    - Small difference: small power
    - Large difference: large power
  - Alpha level (type I error)
  - Standard deviation/measure of variance
  - SAMPLE SIZE

Sample Size and Power

- An underpowered study: An association may not be "statistically significant" in your sample, even though the association exists in the population
- An overpowered study:
  - You may have collected more data than you needed
  - You may detect "trivial" associations

Sample sizes for confidence intervals around intraclass correlation coefficients

- Early approaches: fixed numbers
  - At least 300 (Nunnaly, JC (1978) *Psychometric Theory*. McGraw-Hill, New York.)
  - At least 200 (Guilford, JP (1956) *Psychometric Methods*, *2nd Edition* McGraw Hill, New York.)
- Big issue: Power should depend on the magnitude of the ICC
  - ICCs near 0 will require larger sample sizes, and larger ICCs require smaller sample sizes
  - Sample size recommendations that don't take magnitude of ICC into account: may overpower or underpower a study.

Confidence intervals around intraclass correlation coefficients

- Statistical idea: use formulas developed for Pearson correlation coefficient for the reliability coefficient
- Transform *r* using Fisher's transformation (because distribution of r is not normal):

$$z'(r) = \frac{1}{2}\ln\frac{(1+r)}{(1-r)}$$

Confidence intervals around ICCs

- Define CI width of interest for r (CI_H)
- Calculate z'(r) for expected r
- Variance of z'(r) is $$1/N-3$$
- CI is $$z'(r) \pm Z_{\alpha/2} \frac{1}{\sqrt{N-3}}$$
- Consider that we need to back transform: width of CI for r is defined as

$$z'(r) - z'(r - CI_H)$$

Confidence intervals around ICCs

- Then we can solve for N in the following:

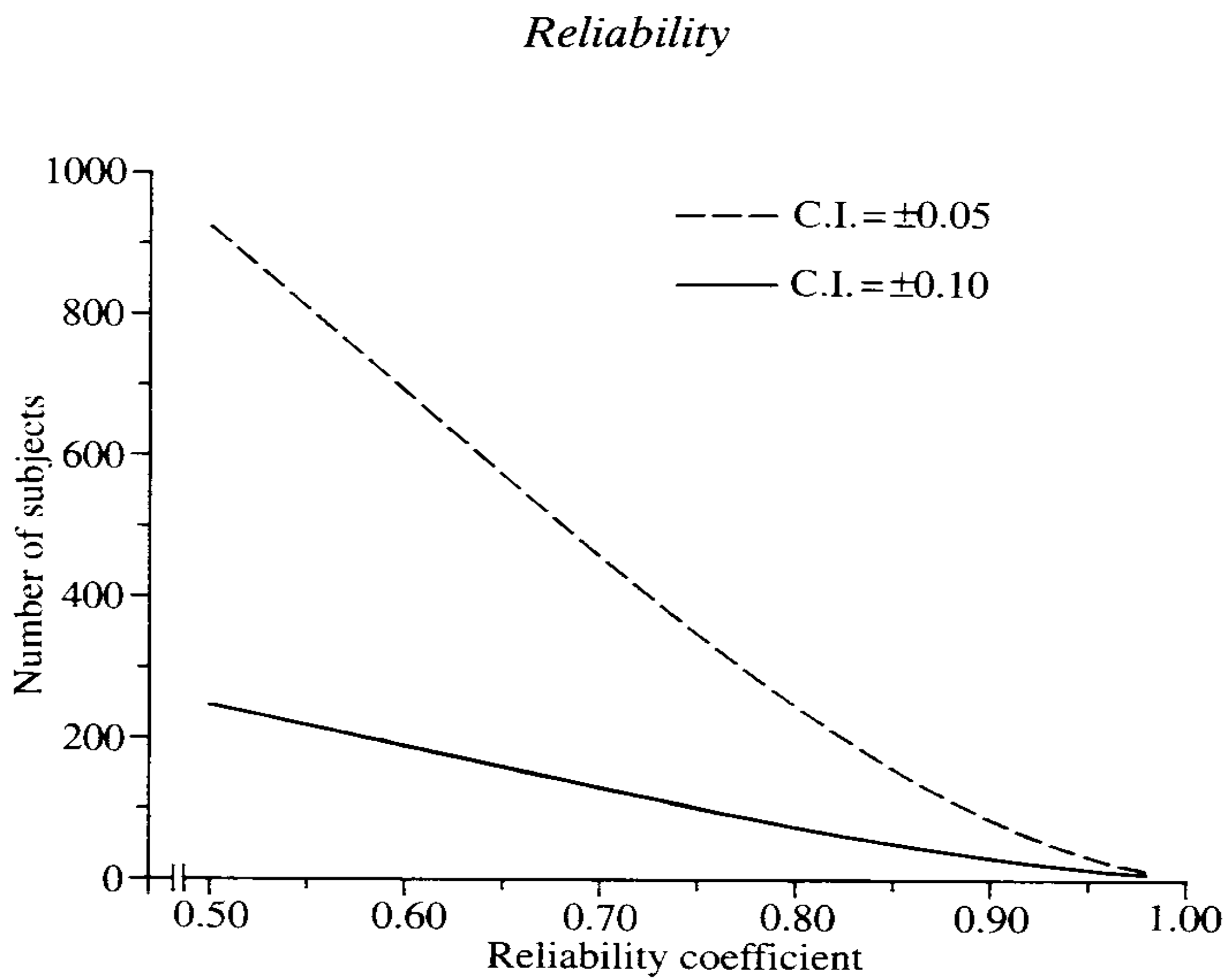
$$z'(r) - z'(r - CI_H) = Z_{\alpha/2} \frac{1}{\sqrt{N-3}}$$

- Therefore, N can be calculated as

$$N = \left[\frac{Z_{\alpha/2}}{z'(r) - z'(r - CI_H)} \right]^2 + 3$$

- TAKE HOME POINT: **<u>N depends on size of r.</u>**

Confidence Intervals around ICCs



Source: Streiner, David L. and Geoffrey R. Norman. 1995. *Health Measurement Scales, 2nd Edition*. New York: Oxford University Press.

Sample size for hypothesis testing with reliability coefficients

- May need to determine if reliability coefficient is significantly different from 0
- May need to determine if reliability coefficient is significantly different from a pre-specified level
  - e.g. is it significantly better than reliability of pre-existing tests?
  - e.g. is reliability of a measure significantly improved by training of raters?

Estimate of sample sizes needed to detect differences between target reliability score (R_1) and a pre-specified reliability score (R_0), with two evaluations per subject.

	R_1								
R_0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	616	152	70	36	22	15	10	7	4
0.1		592	143	61	33	20	12	8	5
0.2			544	129	53	28	16	10	6
0.3				477	109	44	22	11	7
0.4					394	87	33	16	8
0.5						301	63	22	9
0.6							206	40	12
0.7								119	19
0.8									46

Source: Walter, S.D., M. Eliasziw, and A. Donner. 1998. “Sample Size and Optimal Designs for Reliability Studies” *Statistics in Medicine* 17:101-110.

Sample Size for Factor Analysis

- Currently a wide range of recommendations
- OLD Single sample size recommendations
  - Recommendations of at least 100:
    - Gorsuch, R. L. 1983. *Factor Analysis, 2nd edition*. Hillsdale, NJ: Erlbaum
    - Kline, P. 1979. *Psychometrics and Psychology*. London:Academic Press
  - Recommendation of 200:
    - Guilford, J. P. 1954. *Psychometric Methods, 2nd edition*. New York: McGraw-Hill.
  - Recommendation of 250:
    - Cattell, R. B. 1978. *The Scientific Use of Factor Analysis*. New York: Plenum

Sample Size in Factor Analysis

- Single sample size recommendations (cont.)
  - Recommendation of 500:
    - Comrey, A. L., and H. B. Lee. 1992. *A First Course in Factor Analysis*. Hillsdale, NY: Erlbaum
    - These authors also propose a scale, whereby: 100 = poor, 200 = fair, 300 = good, 500= very good, 1000 = excellent

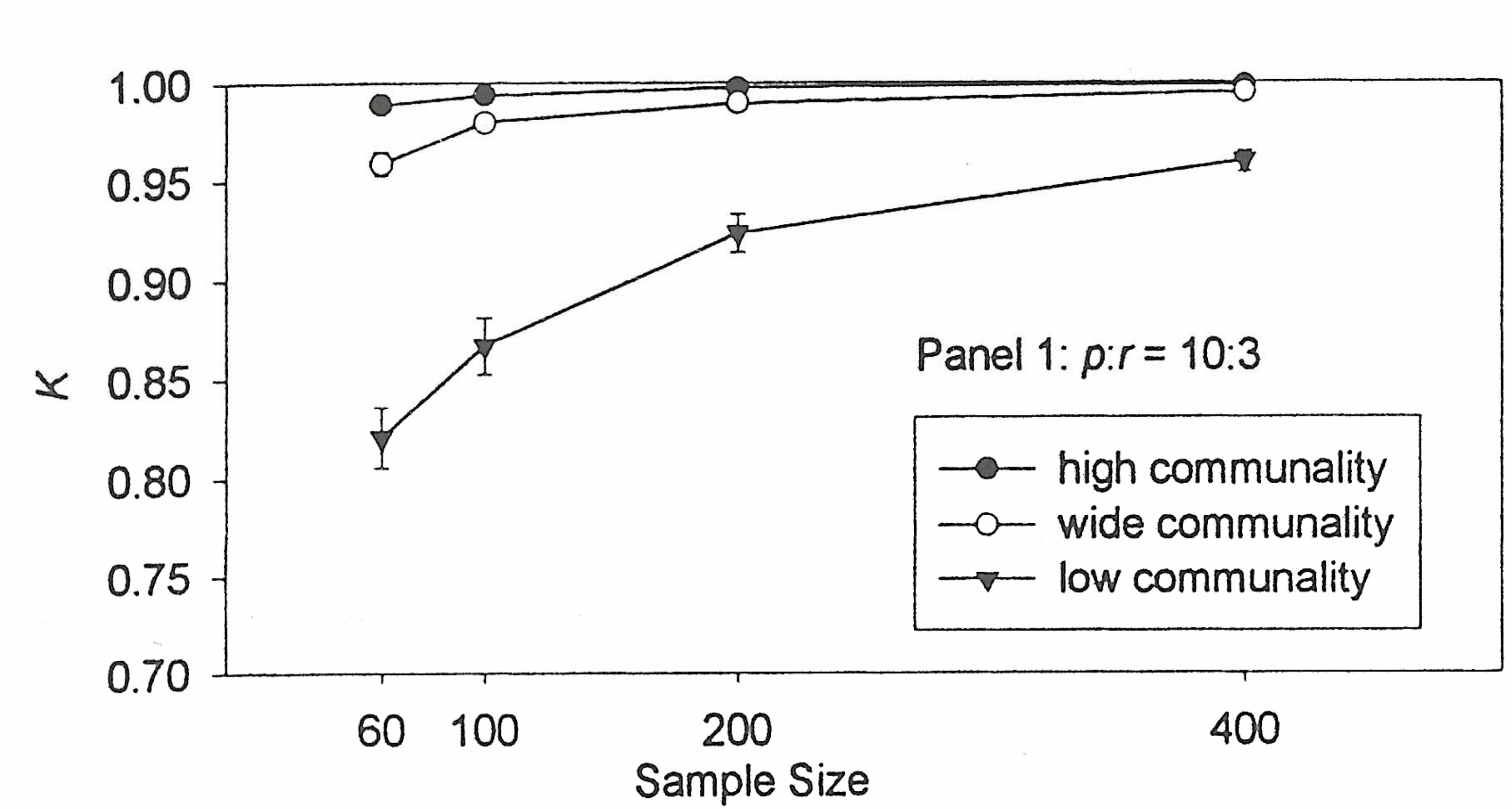
N to p ratio

- N = sample size
- p = number of items included in analysis
- Recommendation of a ratio from 3 to 6: (Cattell, 1978)
- Recommendation of a ratio of at least 5 (Gorsuch, 1983)
- Recommendation of at least 10 (Everitt, BS, 1975).

p:r Ratio -- Modern approach

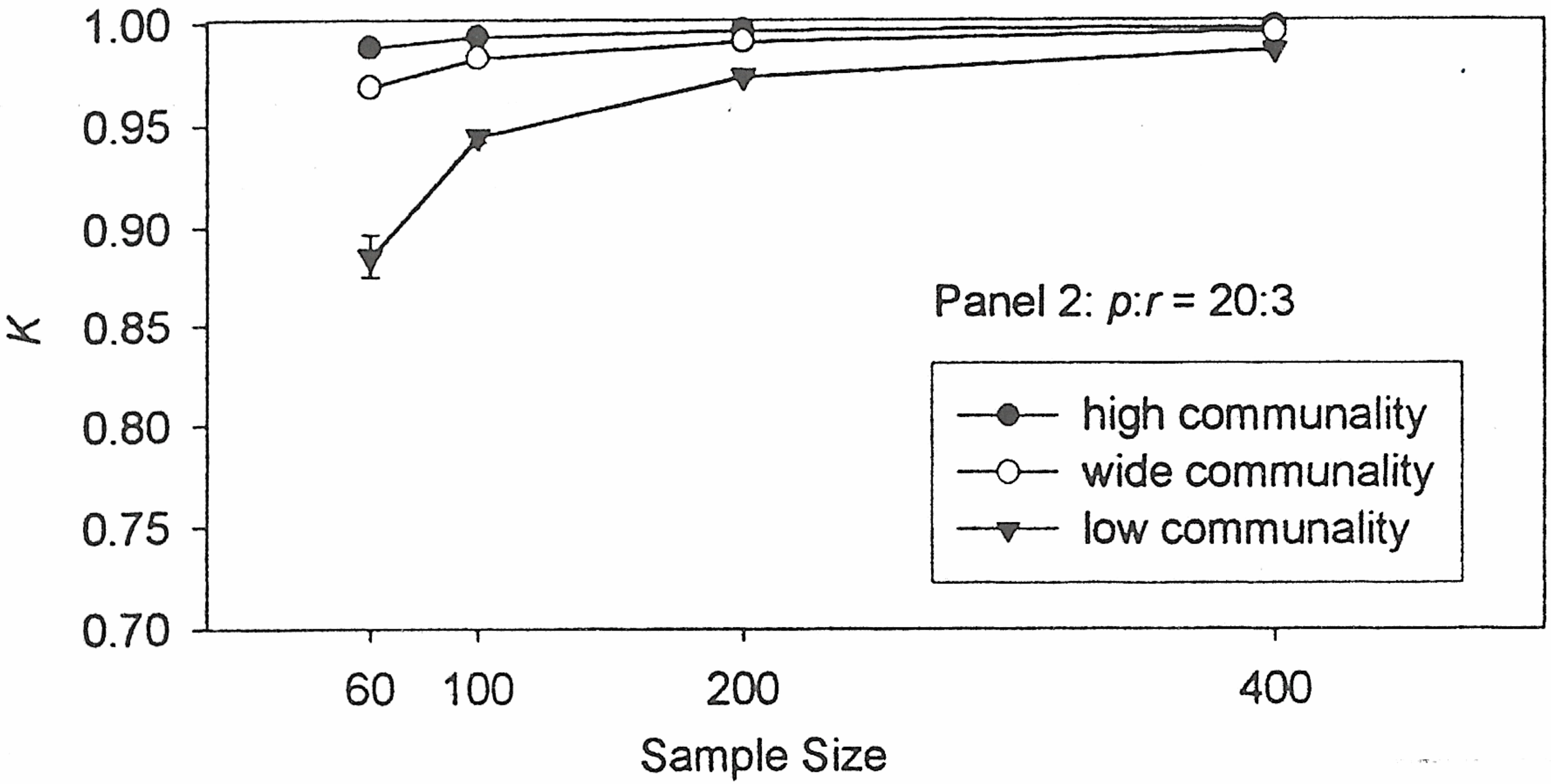
- MacCallum et al approach
- Necessary N, or *N:p* ratio for recovery of population factor is not invariant across studies.
- Simulation studies where samples with N as low as 78 and N:p ratios of 1:3 have resulted in good recovery of population parameters.
- Highlight importance of three factors, in conjunction:
  - Sample size
  - Communality
  - p:r ratio
  - (p=number of items; r=number of factors)

p:r ratio approach



Source: MacCallum, Robert C., Keith F. Widaman, Shaobo Zhang, and Sehee Hong. 1999. “Sample Size in Factor Analysis.” *Psychological Methods* 4:84-99.

p:r ratio approach



Source: MacCallum, Robert C., Keith F. Widaman, Shaobo Zhang, and Sehee Hong. 1999. “Sample Size in Factor Analysis.” *Psychological Methods* 4:84-99.

p:r ratio approach

- Based on definitions for previous graphs:
- *p:r* ratio: number of items to number of factors
- communality:
  - high: 0.6, 0.7, and 0.8
  - wide: 0.2 to 0.8 in 0.1 increments
  - low: 0.2, 0.3, and 0.4

p:r ratio approach

- K: index of interest
- For each factor:

$$\phi_k = \frac{\sum_{j=1}^{p} f_{jk(s)} f_{jk(t)}}{\sqrt{\left(\sum_{j=1}^{p} f_{jk(s)}^2\right)\left(\sum_{j=1}^{p} f_{jk(t)}^2\right)}}$$

where $f_{jk}(t)$ = true population factor loading for variable j on factor k and $f_{jk}(s)$ is the corresponding sample loading

p:r ratio approach

- Summary for the whole model:

$$K = \frac{\sum_{k=1}^{r} \phi_k}{r}$$

- interpretation of K:
  - 0.98 to 1.00 = excellent
  - 0.92 to 0.98 = good
  - 0.82 to 0.92 = borderline
  - 0.68 to 0.82 = poor
  - < 0.68 = terrible

Sample Size Considerations in Latent Class Analysis

- No formal approach has been taken
- Critical factors that will affect ‘necessary’ sample size (in order of importance)
  - Class sizes
    - If trying to detect small classes, need large N
    - Nature of the sample (epidemiologic versus patient population
  - Number of classes to be fit
    - The larger the number of classes, the larger the N

Sample Size Considerations in Latent Class Analysis

- Critical factors that will affect ‘necessary’ sample size (continued)
  - Overall prevalence of items
    - Generally, want items with large variability (close to 0.5)
    - Depends on sensitivity of items (i.e., class specific prevalences)
  - Number of items
    - Some argue need at least 5 per number of possible patterns (doesn’t make sense to me)
    - More items allow more complex structure
    - May tend to increase measurement error
    - More items may tend to ‘encourage’ violation of conditional independence and violate model fit
    - Allows larger number of classes (due to identifiability)