$$\frac{\partial Q}{\partial \beta_0} = -2\Sigma Y_i + 2\beta_1 \Sigma X_i + 2N\beta_0$$

$$\frac{\partial Q}{\partial \beta_1} = -2\Sigma X_i Y_i + 2\beta_0 \Sigma X_i + 2\beta_1 \Sigma X_i^2$$

$\hat{\beta}_0$:
$$-2\Sigma Y_i + 2\hat{\beta}_1 \Sigma X_i + 2N\hat{\beta}_0 = 0$$
$$2N\hat{\beta}_0 = 2\Sigma Y_i - 2\hat{\beta}_1 \Sigma X_i \quad \text{(divide by } 2N\text{)}$$
$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$\hat{\beta}_1$:
$$-2\Sigma X_i Y_i + 2\hat{\beta}_0 \Sigma X_i + 2\hat{\beta}_1 \Sigma X_i^2 = 0$$
$$2\hat{\beta}_1 \Sigma X_i^2 = 2\Sigma X_i Y_i - 2\hat{\beta}_0 \Sigma X_i$$
$$\hat{\beta}_1 \Sigma X_i^2 = \Sigma X_i Y_i - \hat{\beta}_0 \Sigma X_i \quad \text{(substitute in } \hat{\beta}_0\text{)}$$
$$\hat{\beta}_1 \Sigma X_i^2 = \Sigma X_i Y_i - (\bar{Y} - \hat{\beta}_1 \bar{X}) \Sigma X_i$$
$$\hat{\beta}_1 \Sigma X_i^2 = \Sigma X_i Y_i - \bar{Y}\Sigma X_i + \hat{\beta}_1 \bar{X} \Sigma X_i$$

$$\hat{\beta}_1 \Sigma X_i^2 - \hat{\beta}_1 \bar{X} \Sigma X_i = \Sigma X_i Y_i - \bar{Y} \Sigma X_i$$
$$\hat{\beta}_1 \Sigma X_i^2 - \hat{\beta}_1 N \bar{X}^2 = \Sigma X_i Y_i - N\bar{X}\bar{Y}$$
$$\hat{\beta}_1 (\Sigma X_i^2 - N\bar{X}^2) = \Sigma X_i Y_i - N\bar{X}\bar{Y}$$
$$\hat{\beta}_1 = \frac{\Sigma X_i Y_i - N\bar{X}\bar{Y}}{\Sigma X_i^2 - N\bar{X}^2}$$
$$\therefore \quad \hat{\beta} = \frac{\Sigma (X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma (X_i - \bar{X})^2}$$

and $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$