# 57 \[ A^2 (P^2 - 36) \]

\[ F_i (\theta + \alpha) + F_i (\theta - \alpha) = 1 \]

\[ F_1 = F_2 = \ldots = F_h = F \]

\[ h = 15, \alpha = 0.076 \]

Table A.4 \[ \chi^2_{0.076} = 86 \]

Dec. Rule: Reject H₀ if \( T^+ \geq 86 \)
For $\Theta = 1.25$

$(3.18 \& 3.20 \text{ in book})$

\[
A_{\text{nomal}} = \frac{\left(\frac{15\times 1.4}{2} + \frac{15}{\sqrt{2}}\right)}{\sqrt{\frac{15.16.31}{24}}} \times \frac{1.25}{\sqrt{\pi}} - \ldots$

\[
= 1.16 - 1.43 = -0.27
\]

\[
\therefore \text{Approximate Power at } \Theta = \Phi(-0.27)
\]

\[
= 0.3936
\]
\( \alpha = 0.05 \)

\( H_0: \theta = 0 \quad \text{vs} \quad \theta > 0 \)

\[ T^+ / \eta \sim \text{as in 3.21 \textit{(Book)}} \]

\[ p \]

2, \sim F \] identically

2, \sim F

\[ \eta = p(2, + 2 > 0) \quad \text{[Def]} \]

\[ = p(2 > -2) \]

(\text{alternative to the Null})
\[ h = \frac{(2\alpha + 2\beta)}{3(\eta - \frac{1}{2})} \quad \eta > \frac{1}{2} \]

\[ \eta = P(2_1 + 2_2 > 0) \]

\[ = \frac{1}{2} \left( ? \right) \Rightarrow \text{Null} \]

\[ \eta > \frac{1}{2} \rightarrow \text{alternative} \]

[If \( X \sim F, Y \sim F \), \( P(X > Y) = ? \)]

[Assume certain results!!]

\[ P(X > -\frac{1}{2}) = \frac{1}{2} \]
\[ \therefore \alpha = 0.05, \ 2\alpha = 1.645 \]

\[ 1 - \beta \geq 0.84 \]

\[ \Rightarrow \beta \geq 1 - 0.84 = 0.16 \]  
\[ \therefore \text{(Check)} \]

Then find \[ 2\beta = 2 \cdot 0.16 = 0.995 \]

\[ (2\alpha + 2\beta)^2 \]

\[ \therefore \eta \sim \frac{3}{(\eta - \frac{1}{2})^2} = \ldots = 58.08 \]

\[ 95\% < i = 100(1 - \alpha)\% \sim 59 \]
The "Hodges-Lehman" estimator is associated with the WSRT.

To estimate $\Theta$

We first determine quantities that looks like $Z_i + 2Z_j$; $i \leq j = 1(1) n$

There are $\frac{n(n+1)}{2}$ of these.
The quantity \( \frac{2i + 2j}{2} \) is called "Welsh average."
The H-L estimator of $\Theta$, associated with WSRT ($T^+$)
is given by
$\hat{\Theta} = \text{Median} \left\{ \frac{2i + 2j}{2}, i \leq j \right\}_{i=1}^{(1)n}$
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Sign Test (Not WSR Test)

Focus on assumption

\[ F_i(\theta + \alpha) + F_i(\theta - \alpha) = 1 \text{ (WSR)} \]

\[ F_i(\theta) = 1 - F_i(\theta) \text{ (ST)} \]

\[ F_i(\theta) = p(2\alpha \leq \theta) = p(2\alpha > \theta) \]
$$y_i = \begin{cases} 1 & ; \ z_i > 0 \\ 0 & ; \ z_i < 0 \end{cases}$$

Set $B = \sum_{i=1}^{n} y_i$

Test statistic

Look (3.40), (3.41), & (3.42)
Large sample results

\[ E_o(B) = \frac{n}{2} (?) \]
\[ V_o(B) = \frac{n}{4} (?) \]

\( X \sim \text{Bin}(n, p) \) & \( p = \frac{1}{2} \)

\[ E(X) = n \cdot p \]
\[ V(X) = n \cdot p(1-p) \]
\[ n = 25, \ p = 0.5 \]

\[ b_{0.0216, \ 1/2} = 18 \]

Rej \ H_0 if \ \( B \geq 18 \)

\[ B = 21 \rightarrow \text{so reject} \]

\[ P_0 (B \geq 21) = 0.00005 \]