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Bifurcation of vortices in the light-sensitive oscillatory Belousov–Zhabotinsky medium

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Abstract

Experiments were carried out to study the effect of local disturbances on the vortex dynamics in the light-sensitive Belousov–Zhabotinsky (BZ) reaction. It was found that irradiation with visible light of the core region of a rotating spiral wave (vortex) in an oscillatory medium results in a bifurcation which dramatically changes the shape of the vortex. The bifurcation is seen as an unfolding increase in the vortex wavelength although only a negligible change in the vortex core size is observed. Similar dynamics is expected to occur in oscillatory media of different nature.

1. Introduction

Rotating vortices are observed in various excitable and oscillatory media, including surface catalysis on Pt [1], myocardium [2], retinae [3], cultures of microorganisms [4] and single cells [5]. The general aspects of vortex dynamics can be understood if one studies the dynamics in a sample chemical medium such as the Belousov–Zhabotinsky (BZ) reaction. A light-sensitive ruthenium-complex-catalyzed BZ reaction was recently used to study the effect of entrainment and resonance of spiral waves [6]. In that work the entire medium was irradiated with modulated light. In the present study we have used a similar medium, but the irradiation area only included a small part of the medium (the core region of the vortex). Surprisingly, the local irradiation invoked a global sharp change of the shape of vortex. The new shape was different from the Archimedian spiral, which is conventionally assumed to be the unique shape of chemical vortices.

Actually, analytical investigations have shown that in excitable media the shape of vortices, i.e. the location of the front, is an Archimedian spiral if measured far from the core [7,8]. In the case of a vortex with a circular core, the core diameter d_c is related to the wavelength λ:

$$d_c = \alpha \lambda / \pi, \quad (1)$$

with the factor α around 1. It is widely believed that vortices show similar dynamics in oscillatory media, provided the rotation frequency is much higher than the frequency of bulk oscillations. Since the discovery of chemical vortices [9,10] the Archimedian spiral has been the only shape experimentally observed in non-linear chemical systems. Below we present the results of monitoring vortices whose periods are

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comparable to the period of bulk oscillations.

2. Experimental

We carried out the experiments using the light-sensitive BZ reaction mixture of the following composition: 0.5 M $NaBrO_3$, 0.25 M $CH_2(COOH)_2$, 0.25 M H_2SO_4, 2 mM tris(2,2′-bipyridyl)ruthenium (II)chloride, i.e. $Ru(bpy)_3Cl_2$, at a temperature T of 20°C. A probe of 2.43 ml was carefully stirred and poured into a 5.9 cm diameter Petri dish, which was then covered with a glass lid.

We used a 50 mW argon ion laser, wavelength 488 and 514.5 nm, to irradiate the medium. Light irradiation of the BZ reaction led to the production of bromide ions [11–13], which partly suppressed wave conduction. By irradiating the core of the vortex we simply increased its diameter from 0.3 to 0.6 mm. The core of the vortex was the only part of the medium subjected to irradiation.

The dynamics was followed with a video recorder connected to an SGI Indy computer. Simple image processing was performed to measure spatio-temporal characteristics, such as frequency and wavenumber [14].

3. Results

Initially, the vortex is seen as an Archimedian spiral (Fig. 1a) rotating at a frequency 1.33 times as

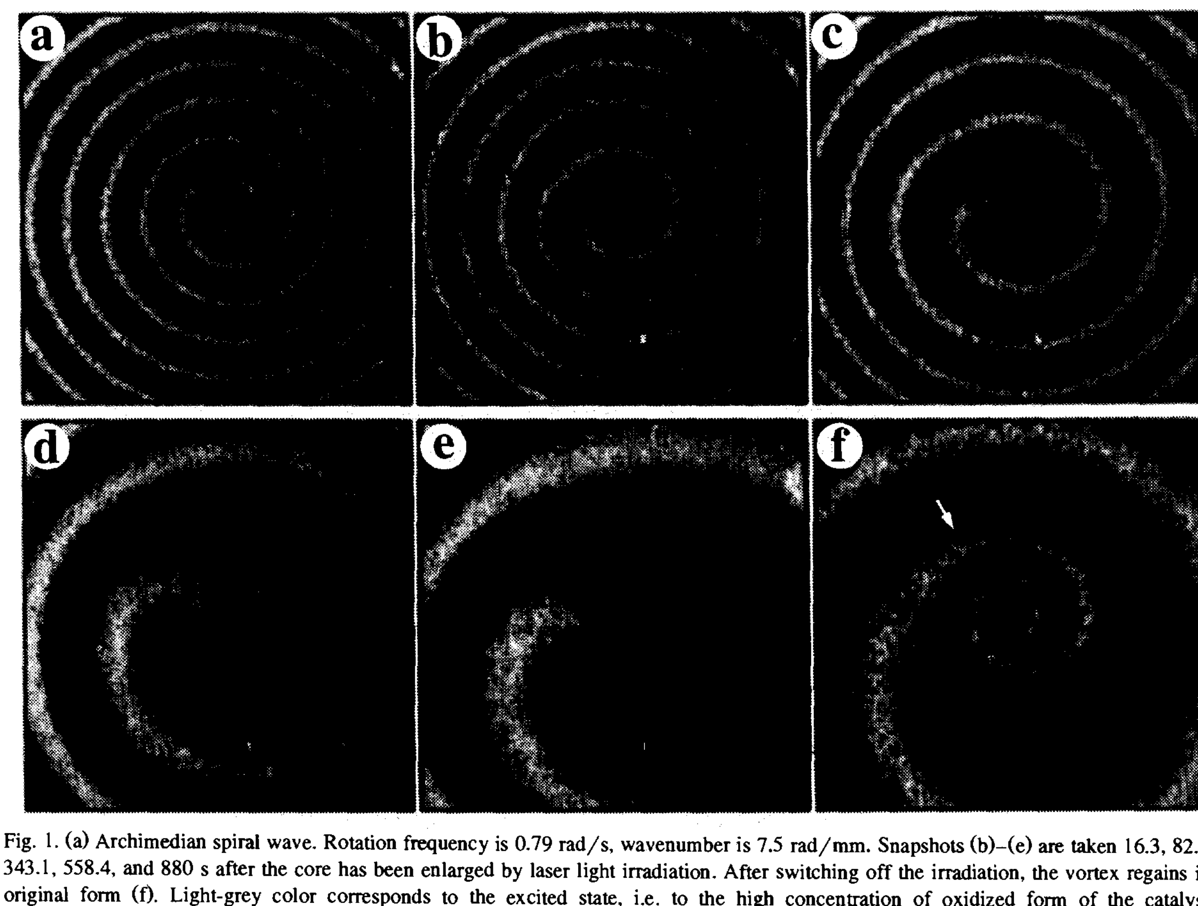

Fig. 1. (a) Archimedian spiral wave. Rotation frequency is 0.79 rad/s, wavenumber is 7.5 rad/mm. Snapshots (b)–(e) are taken 16.3, 82.7, 343.1, 558.4, and 880 s after the core has been enlarged by laser light irradiation. After switching off the irradiation, the vortex regains its original form (f). Light-grey color corresponds to the excited state, i.e. to the high concentration of oxidized form of the catalyst, $Ru(bpy)_3^{3+}$. Note the dramatic change in the shape of the vortex. There is at least a four-fold change in the wavelength and the width of excited state; the change in rotation frequency does not exceed 30%. The size of each snapshot is 73 × 73 mm.

high as that of the bulk oscillations. After the irradiation is switched on, the vortex changes its form; the change is seen as an increase in both the width of the excited state and the wavelength (Fig. 1b–e). There is a four-fold increase in the parameters over a period of 9 min.

To follow the vortex dynamics it is convenient to plot the wavenumber, $k = 2\pi/\lambda$, and the frequency, $\omega = 2\pi/T$ as functions of time (Fig. 2). Note the difference in the evolution of these quantities: after a short transient process ($t = 20–100$ s, Fig. 2a), the frequency is established at a level of 0.6-rad/s, which is experimentally indistinguishable from the frequency of bulk oscillations. Thereafter ($t = 100–800$ s) there is a slow change in frequency which is simply due to reagent aging.

The wavenumber, in contrast, decreases gradually throughout the monitored term (Fig. 2b). Its dynamics can be well approximated by a function $1/(a + bt)$ which tends to zero with time. The decrease in wavenumber indicates that the vortex changes asymptotically to an alternative form, which, in contrast to the Archimedian spiral wave, is termed phase rotor. Obviously, Eq. (1), which is valid for an Archimedian spiral wave, does not hold for the phase rotor whose wavelength λ tends to infinity while d_c is a constant (Fig. 3).

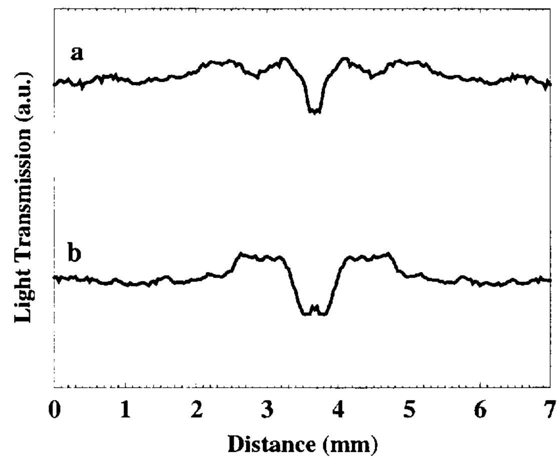

Fig. 3. Profile of the core of a vortex: before (a) and during (b) irradiation. Irradiation results in approximately a two-fold increase in core size. The profiles are obtained after averaging more than 30 images over one period of vortex rotation.

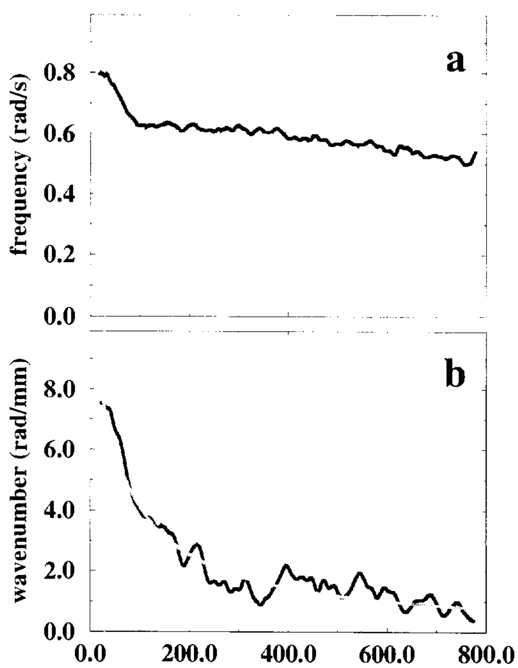

Fig. 2. Vortex frequency (a) and wavenumber (b) monitored at a point 1.7 mm distant from the center of vortex rotation. After the irradiation is switched on at $t = 16.7$ s, the frequency quickly changes from 0.8 to 0.6 rad/s. The wavenumber gradually evolves, tending to zero. Its dynamics is well approximated by the function $1/(a + bt)$ with $a = 0.092$ and $b = 0.0015$ (grey line).

After the irradiation is switched off, the parameters of the vortex revert to their original values and the vortex shape reverts back to the Archimedian spiral (Fig. 1f). In Fig. 1f it is seen that the vortex consists of two clearly distinguished parts. The central part is seen as a two-turn Archimedian spiral, composed of a wave with a short wavelength and a short excited state. At some point (marked in Fig. 1f by an arrow) this wave transforms to a wave with a long wavelength and a wide excited part. It is important to note that the stiffness of the system (the ratio of the period of rotation and the duration of excitation) does not change significantly due to the described transformation of waves; i.e. even in the case of waves having wide excited states (Fig. 1b–f) there are oscillations with a sharp front and a smooth wake at any point.

4. Discussion

To comprehend the observed phenomena we should bear in mind that the Archimedian spiral wave (Fig. 1a, central part of Fig. 1f) has a high frequency, large wavenumber and sharp wavefronts. These are the characteristics of trigger waves. In contrast, the vortex shown in Fig. 1b–e has a frequency close to that of bulk oscillations, the wavenumber tends to zero and the wavefront is smooth. These are the characteristics of phase waves. Thus, the changes in the vortex shape, seen in Fig. 1a–e and Fig. 1f, are closely related to the transition from trigger waves to phase waves and vice versa [15]. Actually, near the core the vortex has a wavenumber $k=2/d_c$; if k in an oscillatory medium becomes below the critical value (inflation point on the dispersion curve [15]), the type of wave switches from a trigger wave to a phase wave. This should result in marked changes in vortex dynamics.

The observed changes can be described as a bifurcation of the vortex shape from Archimedian spiral wave to phase rotor. Formally this bifurcation is a transition between two states: a vortex with a non-zero wavenumber k, and a vortex with a wavenumber $k=0$. The bifurcation parameter is the radius of the vortex core, r_c. Very similar dynamics has been observed in numerical simulations [16]; the transition observed occurred sharply at a fixed value of r_c. That is why we termed the process a bifurcation. Meanwhile, at present there has not been a well-developed theory to classify the bifurcations in distributed systems; the details of the observed phenomena need to be investigated analytically. It should be noted that there is an analytical solution in the form of a non-Archimedian vortex, provided the system possesses small amplitude oscillations near the Hopf bifurcation, the so-called λ–ω system [17–20]. However, these results cannot be applied directly to the large-amplitude oscillations in the system under study.

An interesting subject for study could be the interaction between spiral waves and phase rotors. Preliminary experiments show that spiral waves having a higher frequency dominate in the media, i.e. in the course of time only spiral waves 'survive' in the medium.

Note that in our experiments the core was irradiated by light of high intensity, so that the medium close to the core loses its oscillatory nature. The tip of vortex rotated around this region with a frequency ω indistinguishable from the frequency of bulk oscillations ω_0. The natural question here would be: is the relation $\omega \approx \omega_0$ accidental or not? We believe this relation occurs because after the bifurcation the shape and dynamics of the vortex changed from those of Archimedian spiral wave to phase rotor. The question of whether there is an exact equality or an approximate equality between frequencies cannot be answered in the current work. Further experimental and theoretical investigations are needed.

Irradiation was used in the experiments simply to enlarge the core of the vortex to 0.6 mm. Such a local disturbance triggers the global transition of the vortex shape, which spreads over the entire medium. We believe that it should be possible to observe similar bifurcations of the vortex shape in other media without the application of any external interference. Such media should maintain a vortex with large core and rotation frequency of the vortex should be close to the frequency of bulk oscillations.

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