## Bmtry 763: Tutorial 1

1) Assume a Poisson count model for the data from small areas, with $y_i \sim Poiss(e_i\theta_i)$. The SMR is an estimator of relative risk. For a set of $m$ regions, each region has SMR given by

$$\hat{\theta}_i = y_i/e_i \qquad (1)$$

where $y_i$ is the region disease count and $e_i$ is the expected count.

a) Show that the maximum likelihood estimator of $\theta_i$, $\hat{\theta}_i$ say, is given by (1) above.

b) Show that this estimator has standard error given by

$$se(\hat{\theta}_i) = \sqrt{\theta_i/e_i}$$

c) If $\theta_i = 1,$ show that this standard error only depends on the $e_i$. What happens when $e_i > 1, or \;\; 0 < e_i < 1.$

2) A smoothed estimator of relative risk is given by

$$\hat{\theta}_i = (y_i + a)/(e_i + b)$$

for fixed $a, b$

Show that the standard error of this estimator only depends on $e_i, \theta_i,$ and $b.$

3) Within a study window A, a set (realization) of case events are observed. They are denoted $\{x_i\}$, $i = 1,....,n$. The first order intensity of the Poisson process is

$\lambda(x) = \rho g(x)$.

The log-likelihood for the intensity is given by:

$$l = \sum_i \ln(\lambda(x_i)) - \int_A \lambda(u)du$$

Find the maximum likelihood estimator of $\rho$.