Goodness of Fit

DIC, MSPE, MPL, WAIC notes

Goodness of Fit

- Convergence does not mean a model fits well
- Goodness-of-fit (GOF) measures should be used to compare how well models fit the data

Goodness of Fit (BB, ch 10)

- Various measures are available:
  Deviance:

$$D(\boldsymbol{\theta}) = -2\sum_{i=1}^{m} \log f(y_i, \boldsymbol{\theta}) = -2l(\mathbf{y}, \boldsymbol{\theta})$$

Mean square error (or other residual based measures)

MSE:

$$MSE_y = \frac{1}{m}\sum_{i=1}^{m}\{y_i - \hat{y}_i\}^2$$

where $\hat{y}_i$ could be a fitted value

Fitted values could be computed in different ways

AIC, BIC, DIC: information criterion measures that penalize for number of parameters

Deviance Information Criterion

- DIC is defined as:

$$DIC = \bar{D}(\boldsymbol{\theta}) + pD$$

$$pD = \bar{D}(\boldsymbol{\theta}) - \hat{D}(\boldsymbol{\theta}) = dbar - dhat$$

$\hat{D}(\boldsymbol{\theta})$: deviance evaluated

at posterior estimates of $\boldsymbol{\theta}$

$\bar{D}(\boldsymbol{\theta})$: deviance averaged over sample

Deviance Information Criterion

- Deviance information Criterion (DIC)
  - Smaller better; comparative tool
  - pD measures the effective number of parameters
  - Hence DIC and pD should be small ideally
  - Relative measure only: can have negative DIC .... BUT cant have negative pD

  - In this example (SC congenital deaths 1990) after 20,000 the DIC is 169.7 with pD= 3.19
  - Hence the effective number of parameters is 4
  - Difference of 3-5 in DIC is 'significant' when you fit a series of models

MSPE and MPL

- MSPE is a predictive measure model loss
- PPL is the posterior predictive loss and is based on

$$PPL_i = y_i - y_i^{pred}$$

  - This is the predictive residual
- MSPE is the mean squared error:

$$MSPE = \sum_i PPL_i^2 / m$$

- Note : it is not adjusted for parameterization but can be scaled by transformations.

MPL

- Marginal Penalized Likelihood (MPL)
- This is a leave-one out cross-validation measure
  - can be computed easily from the CPO
  - The MPL can be computed across any model (mixtures or uni-modal)
  - It is not adjusted for parameterization

$$MPL = \sum_i \log(CPO_i)$$

  - Criteria: least negative : better model (on log scale)

WAIC

- A new measure that is now superceding DIC.
- The effective parameter penalty cannot be negative in WAIC

- Formula:

$$\text{WAIC} = \sum_i \log \overline{p}(y_i \mid \theta) - \sum_i \text{var}_{\text{post}}(\log p(y_i \mid \theta))$$

where $\overline{p}$ is the sample average density

- Must be computed outside WinBUGS