Goodness of Fit

DIC, MSPE, MPL, WAIC notes

Goodness of Fit

- Convergence does not mean a model fits well
- Goodness-of-fit (GOF) measures should be used to compare how well models fit the data

Goodness of Fit (BB, ch 10)

Various measures are available:

Deviance:

$$D(\mathbf{\theta}) = -2\sum_{i=1}^{m} \log f(\mathbf{y}_{i}, \mathbf{\theta}) = -2I(\mathbf{y}, \mathbf{\theta})$$

Mean square error (or other residual based

measures)

MŚE:

$$MSE_{y} = \frac{1}{m} \sum_{i=1}^{m} \{y_{i} - \hat{y}_{i}\}^{2}$$

where \hat{y}_i could be a fitted value

Fitted values could be computed in different ways

AIC, BIC, DIC: information criterion measures that penalize for number of parameters

Deviance Information Criterion

DIC is defined as:

$$DIC = \overline{D}(\mathbf{\theta}) + pD$$

$$pD = \overline{D}(\theta) - \hat{D}(\theta) = dbar - dhat$$

 $\hat{D}(\theta)$: deviance evaluated

at posterior estimates of θ

 $\overline{D}(\theta)$: deviance averaged over sample

Deviance Information Criterion

- Deviance information Criterion (DIC)
 - Smaller better; comparative tool
 - pD measures the effective number of parameters
 - Hence DIC and pD should be small ideally
 - Relative measure only: can have negative DIC BUT cant have negative pD
 - In this example (SC congenital deaths 1990) after 20,000 the DIC is 169.7 with pD= 3.19
 - Hence the effective number of parameters is 4
 - Difference of 3-5 in DIC is 'significant' when you fit a series of models

MSPE and MPL

- MSPE is a predictive measure model loss
- PPL is the posterior predictive loss and is based on

$$PPL_i = y_i - y_i^{pred}$$

- This is the predictive residual
- MSPE is the mean squared error:

$$MSPE = \sum_{i} PPL_{i}^{2} / m$$

 Note: it is not adjusted for parameterization but can be scaled by transformations.

MPL

- Marginal Penalized Likelihood (MPL)
- This is a leave-one out cross-validation measure
 - can be computed easily from the CPO
 - The MPL can be computed across any model (mixtures or uni-modal)
 - It is not adjusted for parameterization

$$MPL = \sum_{i} log(CPO_{i})$$

• Criteria: least negative : better model (on log scale)

WAIC

- A new measure that is now superceding DIC.
- The effective parameter penalty cannot be negative in WAIC

• Formula:

WAIC =
$$\sum_{i} log \overline{p}(y_i \mid \theta) - \sum_{i} var_{post}(log p(y_i \mid \theta))$$

where \overline{p} is the sample average density

Must be computed outside WinBUGS