Correlated Heterogeneity on WinBUGS

WinBUGS allows a range of types of random effects to be specified.
The log relative risk can be modeled as:

$$\log \theta_i = x_i'\beta + u_i + v_i$$

Here there is a linear predictor ($x_i'\beta$) that includes the fixed covariates and two random terms (u_i, v_i)
This is sometimes called the convolution model
Usually one term is a correlated effect (u_i say)
and the other is an uncorrelated effect (v_i say)

This model is also known as the BYM model after Besag et al (1991) who first suggested it. It has become the standard random effect model used in disease mapping and can be fitted easily on WinBUGS.

How is the correlated effect modeled?

We use a correlated prior distribution for this.
This prior could take various forms.
(What should the prior do to the rates ?)

However, usually a conditional autoregressive (CAR) prior distribution is assumed as it usually can be fitted easily and usually suffices to describe the correlation. Other more sophisticated forms are possible (also on WinBUGS).

Specification of a CAR prior distribution

The distribution model for the uncorrelated spatial effect is assumed to be

$$v_i \sim \text{normal}(0,\sigma_v^2) \quad \forall i.$$

For the spatially correlated random effect the conditional autoregressive (CAR) model proposed by Besag *et al.* (1991) is used:

$$\left[u_i \middle| u_k, i \neq k, \sigma_u^2\right] \sim \text{normal}\left(\bar{u}_i, \sigma_i^2\right)$$

where

$$\bar{u}_i = \frac{1}{\#\{C_i\}} \sum_{k \in C_i} u_k \,, \quad \sigma_i^2 = \frac{\sigma_u^2}{\#\{C_i\}} \,.$$

C_i denotes the set of neighbours of the spatial location i and $\#\{C_i\}$ the number of spatial locations in C_i.

Parameters σ_v^2 and σ_u^2 control, respectively, the variability of v and u.

```
MODEL
model
{
for (i in 1:m)
{
	# Poisson likelihood for observed counts
	y[i]~dpois(mu[i])
	log(mu[i])<-log(e[i])+alpha+u[i]+v[i]
	# Relative Risk
	theta[i]<-exp(alpha+u[i]+v[i])
	# Posterior probability of RR[i]>1
	PP[i]<-step(theta[i]-1+eps)
	# Prior distribution for the uncorrelated heterogeneity
	v[i]~dnorm(0,tau.v)
}

eps<-1.0E-6

# CAR prior distribution for spatial correlated
heterogeneity
u[1:m]~car.normal(adj[],weights[],num[],tau.u)

# Weights
for(k in 1:sumNumNeig)
{
	weights[k]<-1
}

# Improper prior distribution for the mean relative risk in
the study region
alpha~dflat()
```

mean<-exp(alpha)

Hyperprior distributions on inverse variance parameter of random effects
tau.u~dgamma(0.5,0.0005)
tau.v~dgamma(0.5,0.0005)

}

DATA South Carolina: deaths from congenital abnormalities, 1990

INITS initial values: chain 1

INITS initial values for chain 2